Strong and extensional stability

through the prism of dialogue games

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Journées en l'honneur de Thomas Ehrhard
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First encounter: a life-changing talk in Amsterdam (1991)

The first time I heard Thomas talk about his work with Antonio at LICS was a powerful experience that left a lasting impression.

Sequentiality and Strong Stability

Antonio Bucciarelli
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Università di Pisa, Italy
and
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Ecole Normale Supérieure, France
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Thomas Ehrhard
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Université Paris VII, France
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The paper is sharp as a knive, an elegant and creative mathematical work, and a source of inspiration to further explore the structures of sequentiality.

Hypercoherence spaces (1993)

Thomas then discovers how to « linearize » the strongly stable model by refining coherence spaces into hypercoherence spaces!

Hypercoherences: a strongly stable model of linear logic

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Abstract

We present a model of classical linear logic based on the notion of *strong stability* that was introduced in [BE], a work about sequentiality written jointly with Antonio Bucciarelli.

The golden age of linearization

Quite extraordinarily, the main qualitative (= finitary) models of

PCF = simply-typed λ-calculus + arithmetics + fixpoint operator

are refined into models of linear logic:

stability (Berry 1978)	$\rightsquigarrow$	**coherence spaces** (Girard 1987)
strong stability (Bucciarelli-Ehrhard, 1991)	$\rightsquigarrow$	**hypercoherence spaces** (Ehrhard 1993)
sequential algorithms (Berry-Curien 1982)	$\rightsquigarrow$	**sequential data structures** (Lamarche, Curien 1993)
extensional stability (Berry 1979)	$\rightsquigarrow$	**bistructures** (Curien-Plotkin-Winskel, 2000)

Extensional collapse of sequential algorithms (1996)

A few years later, Thomas establishes that the strongly stable model is the « extensional collapse » of the sequential algorithm model.

ELSEVIER

Annals of Pure and Applied Logic 77 (1996) 201–244

ANNALS OF PURE AND APPLIED LOGIC

Projecting sequential algorithms on strongly stable functions

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Received 5 December 1992; revised 12 May 1995; Communicated by J.-Y. Girard

A trail of audacious and inspirational works

- **serial and parallel hypercoherence spaces** with Pierre Boudes
- **indexed linear logic and intersection types** with Antonio Bucciarelli
- **differential lambda-calculus** with Laurent Regnier
- **probabilistic coherence spaces** with Vincent Danos
- **encoding the π-calculus in differential nets** with Olivier Laurent
- **probabilistic PCF** with Michele Pagani and Christine Tasson
- **the free exponential modality of probabilistic coherence spaces** with Raphaëlle Crubillé, Michele Pagani and Christine Tasson
- **linear logic with fixpoints** with Farzad Jafar-Rahmani
- more recently: **coherent differentiation**

Game semantics and relational semantics

One important open question was to understand the connection between

- ▷ **dynamic semantics:** sequential algorithms, game semantics, etc.
- ▷ **static semantics:** relational semantics, coherence spaces, etc.

Patrick Baillot, Vincent Danos, Thomas Ehrhard, Laurent Regnier
Timeless Games.
Proceedings of Computer Science Logic (CSL) 1997

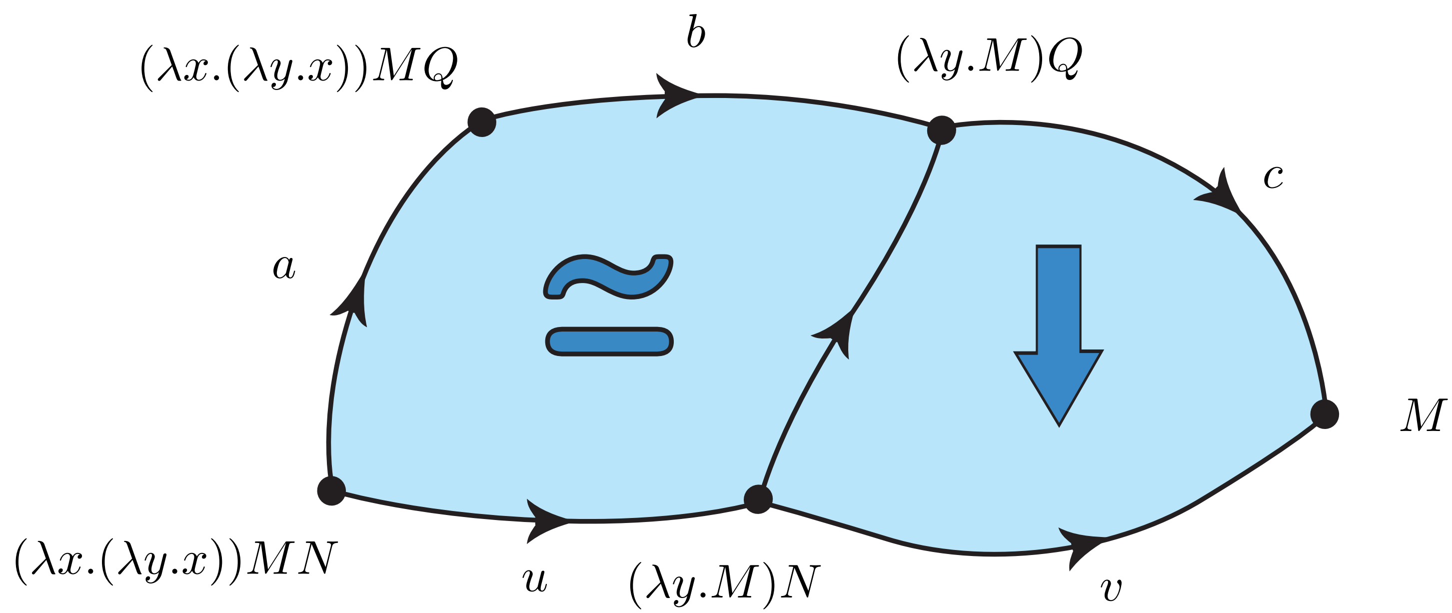
Martin Hyland and Andrea Schalk
Abstract Games for Linear Logic.
Electronic Notes in Theoretical Computer Science, 1999.

Thomas Ehrhard
Parallel and serial hypercoherences.
Theoretical Computer Science 2000

Pierre Boudes
Projecting Games on Hypercoherences.
Proceedings of ICALP 2004

An insight coming from Huet and Lévy

In rewriting theory, it is important to consider rewriting paths modulo redex permutations of the form



Hypercoherence spaces
as the starting point of an exciting journey

In order to connect sequential games and hypercoherence spaces,
it appears necessary to **play on directed acyclic graphs** instead of trees.

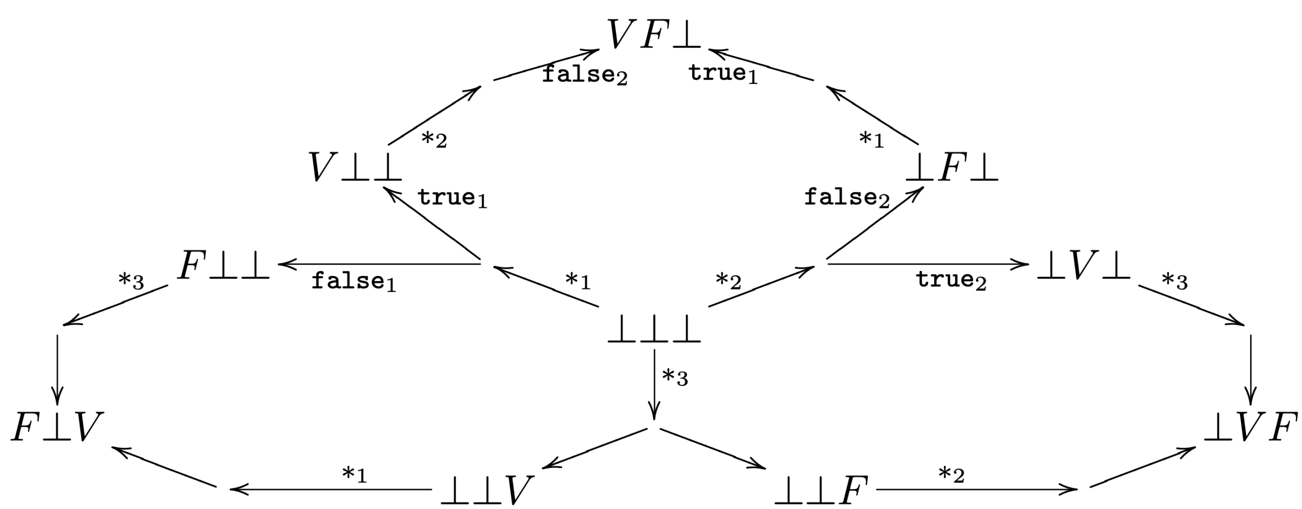


Fig. 1. A fragment of the extensional data structure $!\mathbb{B}\otimes!\mathbb{B}\otimes!\mathbb{B}$.

Excerpt from PAM, Sequential algorithms and strongly stable functions, TCS 2005.

Tensorial logic

A logic of linear continuations

Starting point

We explain in what sense

game semantics is a **syntax** of continuations

and exploit this idea to interpret

dialogue games	$\mapsto$	hypercoherence spaces
innocent strategies	$\mapsto$	cliques

Continuations

Captures the difference between addition as a **function**

$$\mathtt{nat} \times \mathtt{nat} \quad \Rightarrow \quad \mathtt{nat}$$

and addition as a **sequential algorithm**

$$(\mathtt{nat} \Rightarrow \bot) \Rightarrow \bot \quad \times \quad (\mathtt{nat} \Rightarrow \bot) \Rightarrow \bot \quad \times \quad (\mathtt{nat} \Rightarrow \bot) \quad \Rightarrow \quad \bot$$

This enables one to distinguish the **left-to-right** implementation

$$\mathtt{lradd} \quad = \quad \lambda\varphi.\, \lambda\psi.\, \lambda k.\, \varphi\,(\,\lambda x.\, \psi\,(\,\lambda y.\, k\,(x + y))\,)$$

from the **right-to-left** implementation

$$\mathtt{rladd} \quad = \quad \lambda\varphi.\, \lambda\psi.\, \lambda k.\, \psi\,(\,\lambda y.\, \varphi\,(\,\lambda x.\, k\,(x + y))\,)$$

The left-to-right addition

$\neg\neg$ `nat`	$\times$	$\neg\neg$ `nat`	$\Rightarrow$	$\neg\neg$ `nat`
				`question`
`question`				
12				
		`question`		
		5		
				17

$$\texttt{lradd} \quad = \quad \lambda\varphi.\,\lambda\psi.\,\lambda k.\,\varphi\,(\,\lambda x.\,\psi\,(\,\lambda y.\,k\,(x+y))\,)$$

The right-to-left addition

$\neg\neg$ nat $\times$ $\neg\neg$ nat $\Rightarrow$ $\neg\neg$ nat

question

question

5

question

12

17

$$\texttt{rladd} = \lambda\varphi.\lambda\psi.\lambda k.\psi\,(\lambda y.\varphi\,(\lambda x.k\,(x+y)))$$

Tensorial logic

tensorial logic = a logic of tensor, sum and negation

= linear logic without $A \cong \neg\neg A$

= the syntax of linear continuations

= the syntax of dialogue games

= another way to speak about « ludics »

Tensorial logic

▷ Formulas

$$A, B ::= \quad 0 \mid 1 \mid A \oplus B \mid A \otimes B \mid \neg A$$

▷ Sequents

$$A_1 , \ldots , A_k \quad \vdash \quad B$$

⟶ where each hypothesis A_i is a formula

⟶ where the conclusion B is either a formula or the constant $\bot$.

Tensorial logic

Axiom $$\frac{}{A \vdash A}$$

$$\frac{\Gamma \vdash A \qquad A\,,\,\Delta \vdash B}{\Gamma\,,\,\Delta \vdash B}$$ Cut

Left $\neg$ $$\frac{\Gamma \vdash A}{\Gamma\,,\,\neg A \vdash \bot}$$

$$\frac{\Gamma\,,\,A \vdash \bot}{\Gamma \vdash \neg A}$$ Right $\neg$

Exchange $$\frac{A_1\,,\,\ldots\,,\,A_k \vdash B}{A_{\sigma(1)}\,,\,\ldots\,,\,A_{\sigma(k)} \vdash B}$$ for any permutation σ

Tensorial logic

Left $\otimes$ $$\frac{\Gamma, A, B \vdash C}{\Gamma, A \otimes B \vdash C}$$

$$\frac{\Gamma \vdash A \quad \Delta \vdash B}{\Gamma, \Delta \vdash A \otimes B}$$ Right $\otimes$

Left 1 $$\frac{\Gamma \vdash A}{\Gamma, 1 \vdash A}$$

$$\frac{}{\vdash 1}$$ Right 1

Tensorial logic

Right $\oplus_L$

$$\frac{\Gamma \vdash A}{\Gamma \vdash A \oplus B}$$

Left $\oplus$

$$\frac{\Gamma, A \vdash C \quad \Gamma, B \vdash C}{\Gamma, A \oplus B \vdash C}$$

Right $\oplus_R$

$$\frac{\Gamma \vdash B}{\Gamma \vdash A \oplus B}$$

Left 0

$$\frac{}{\Gamma, 0 \vdash A}$$

Right 0

no introduction rule

Left-to-right implementation

$$\dfrac{\dfrac{}{A \vdash A} \qquad \dfrac{}{B \vdash B}}{A, B \vdash A \otimes B}\ \text{Right } \otimes$$

$$\dfrac{A, B \vdash A \otimes B}{B, \neg (A \otimes B), A \vdash}\ \text{Left } \neg$$

$$\dfrac{B, \neg (A \otimes B), A \vdash}{\neg (A \otimes B), A \vdash \neg B}\ \text{Right } \neg$$

$$\dfrac{\neg (A \otimes B), A \vdash \neg B}{A, \neg\neg B, \neg (A \otimes B) \vdash}\ \text{Left } \neg$$

$$\dfrac{A, \neg\neg B, \neg (A \otimes B) \vdash}{\neg\neg B, \neg (A \otimes B) \vdash \neg A}\ \text{Right } \neg$$

$$\dfrac{\neg\neg B, \neg (A \otimes B) \vdash \neg A}{\neg (A \otimes B), \neg\neg A, \neg\neg B \vdash}\ \text{Left } \neg$$

$$\dfrac{\neg (A \otimes B), \neg\neg A, \neg\neg B \vdash}{\neg\neg A, \neg\neg B \vdash \neg\neg (A \otimes B)}\ \text{Right } \neg$$

$$\dfrac{\neg\neg A, \neg\neg B \vdash \neg\neg (A \otimes B)}{\neg\neg A \otimes \neg\neg B \vdash \neg\neg (A \otimes B)}\ \text{Left } \otimes$$

$$\texttt{lrsched} \quad = \quad \lambda\varphi.\, \lambda\psi.\, \lambda k.\, \varphi\,(\,\lambda x.\, \psi\,(\,\lambda y.\, k\,(x, y))\,)$$

Right-to-left implementation

$$
\begin{array}{c|l}
\hline
A \vdash A \qquad B \vdash B & \\
\hline
A , B \vdash A \otimes B & \text{Right } \otimes \\
\hline
A , B , \neg (A \otimes B) \vdash & \text{Left } \neg \\
\hline
B , \neg (A \otimes B) \vdash \neg A & \text{Right } \neg \\
\hline
B , \neg (A \otimes B) , \neg\neg A \vdash & \text{Left } \neg \\
\hline
\neg (A \otimes B) , \neg\neg A \vdash \neg B & \text{Right } \neg \\
\hline
\neg (A \otimes B) , \neg\neg A , \neg\neg B \vdash & \text{Left } \neg \\
\hline
\neg\neg A , \neg\neg B \vdash \neg\neg (A \otimes B) & \text{Right } \neg \\
\hline
\neg\neg A \otimes \neg\neg B \vdash \neg\neg (A \otimes B) & \text{Left } \otimes
\end{array}
$$

$$\texttt{rlsched} \quad = \quad \lambda\varphi.\, \lambda\psi.\, \lambda k.\, \psi\,(\, \lambda y.\, \varphi\,(\, \lambda x.\, k\,(x, y))\,)$$

Dialogue categories

The categorical counterpart of game semantics

Dialogue categories

Definition.

A dialogue category $\mathcal{D}$ is a symmetric monoidal category with

- an object $\bot$
- a functor

$$A \mapsto \neg A \quad : \quad \mathcal{D}^{op} \longrightarrow \mathcal{D}$$

- a family of bijections

$$\varphi_{A,B} \quad : \quad \mathcal{D}(A \otimes B, \bot) \longrightarrow \mathcal{D}(A, \neg B)$$

  natural in A and B.

Dialogue categories

Definition. A dialogue category $\mathcal{D}$ has **finite sums** when

- ▷ the underlying category $\mathcal{D}$ has finite sums
- ▷ the finite sums distribute over the tensor product.

This means that the canonical morphisms

$$(A \otimes B) \oplus (A \otimes C) \longrightarrow A \otimes (B \oplus C) \qquad\qquad 0 \longrightarrow A \otimes 0$$

are isomorphisms.

The free dialogue category $\mathcal{F}$ with sums

▷▷ The objects of $\mathcal{F}$ are the **formulas** of tensorial logic

$$A, B ::= \quad 0 \mid 1 \mid A \oplus B \mid A \otimes B \mid \neg A$$

modulo a series of equations:

- ▷ associativity of $\oplus$ and $\otimes$
- ▷ the unit laws of $\oplus$ and $\otimes$
- ▷ distributivity of $\oplus$ over $\otimes$.

▷▷ The morphisms of $\mathcal{F}$ are the **derivation trees** of tensorial logic modulo a series of equations.

The free dialogue category $\mathcal{F}$ with sums

Fact. Every dialogue category with sums $\mathcal{D}$ induces a functor

$$[-] \quad : \quad \mathcal{F} \longrightarrow \mathcal{D}$$

which preserves the logical structure:

- ▷ negation
- ▷ finite sums $\oplus$ and the unit 0
- ▷ tensor products $\otimes$ and the unit 1

Here, we follow the philosophy of Jim Lambek

Dialogue games

A symmetrized notion of concrete data structure

The free dialogue category $\mathcal{F}$ with sums

Key theorem.

▷▷ The objects of $\mathcal{F}$ are the **dialogue games** generated by

$$A, B ::= \quad 0 \mid 1 \mid A \oplus B \mid A \otimes B \mid \neg A$$

▷▷ The morphisms of $\mathcal{F}$ are **total** and **innocent strategies**.

Innocent strategies are the « proof-nets » of game semantics

The correspondence

Fact. Every formula of tensorial logic

$$A, B ::= \quad 0 \mid 1 \mid A \oplus B \mid A \otimes B \mid \neg A$$

modulo the equations:

- ▷ associativity of $\oplus$ and $\otimes$
- ▷ the unit laws of $\oplus$ and $\otimes$
- ▷ distributivity of $\oplus$ over $\otimes$

is uniquely determined by a normal form

$$A \quad = \quad \bigoplus_{i \in I} \bigotimes_{j \in J_i} \neg A_{ij}$$

The correspondence

This canonical decomposition should be interpreted as follows:

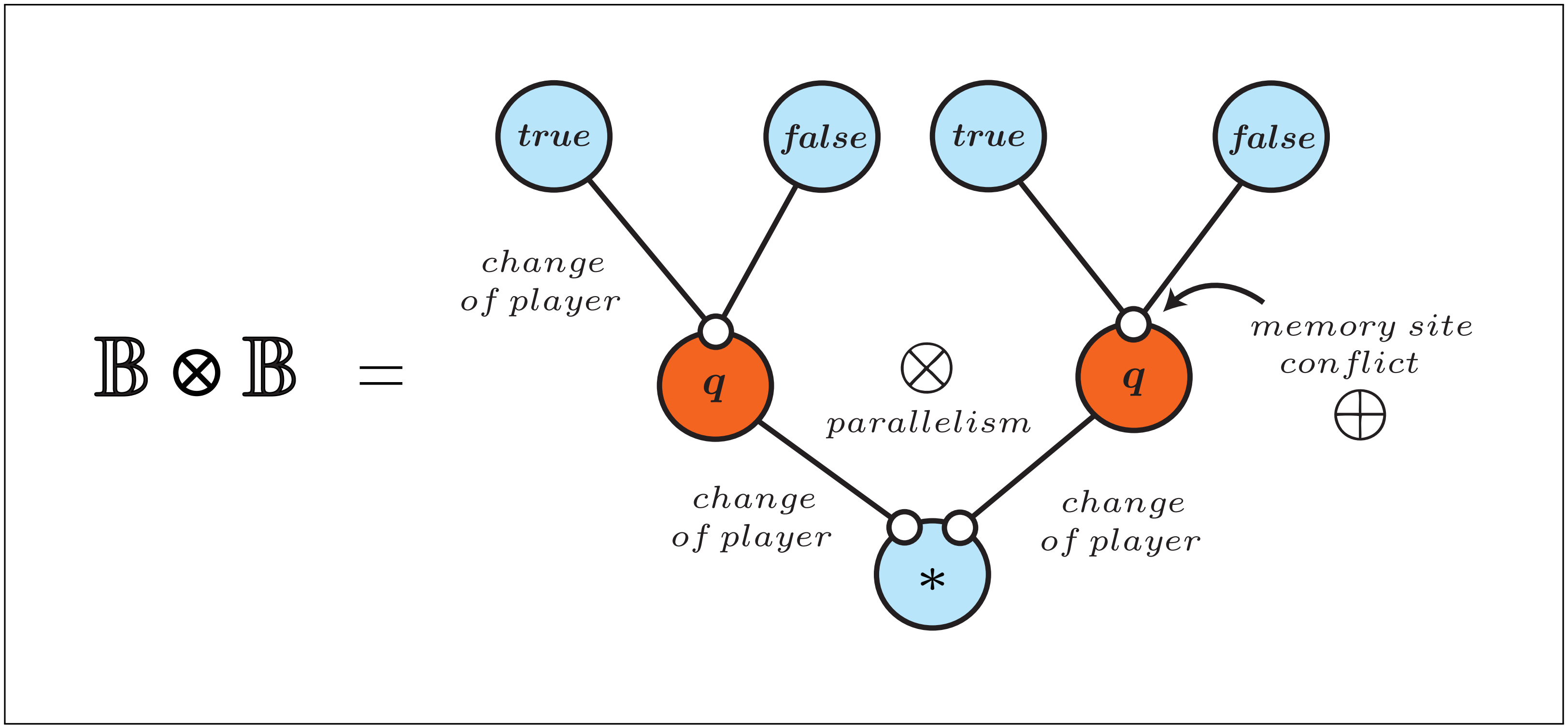
$$A = \bigoplus_{i\in I} \bigotimes_{j\in J_i} \neg A_{ij}$$

$\bigoplus$: *conflict*

$\bigotimes$: *parallelism*

$\neg$: *change of player*

The correspondence



Dialogue games

Definition. A rooted dialogue game is a bipartite tree

$$(C, V, \triangleright)$$

with nodes separated into a set C of **cells** and a set V of **values**.
By bipartite, one means that

$$\triangleright \quad \subseteq \quad C \times V \; + \; V \times C.$$

This tree is moreover equipped with a polarity function

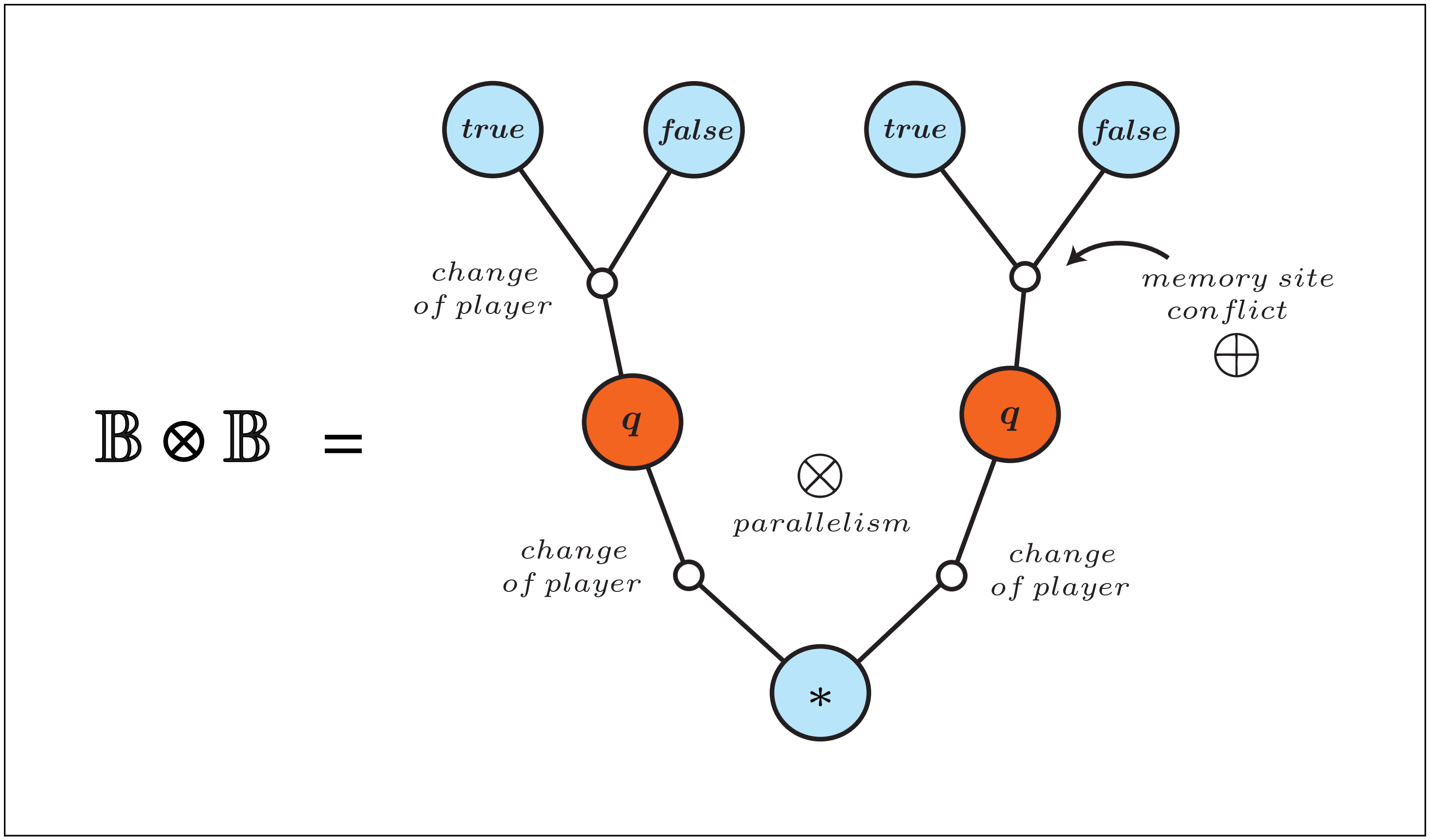
$$\lambda \quad : \quad C + V \quad \longrightarrow \quad \{+1, -1\}$$

such that for every cell $\alpha \in C$ and every value $v \in V$ one has:

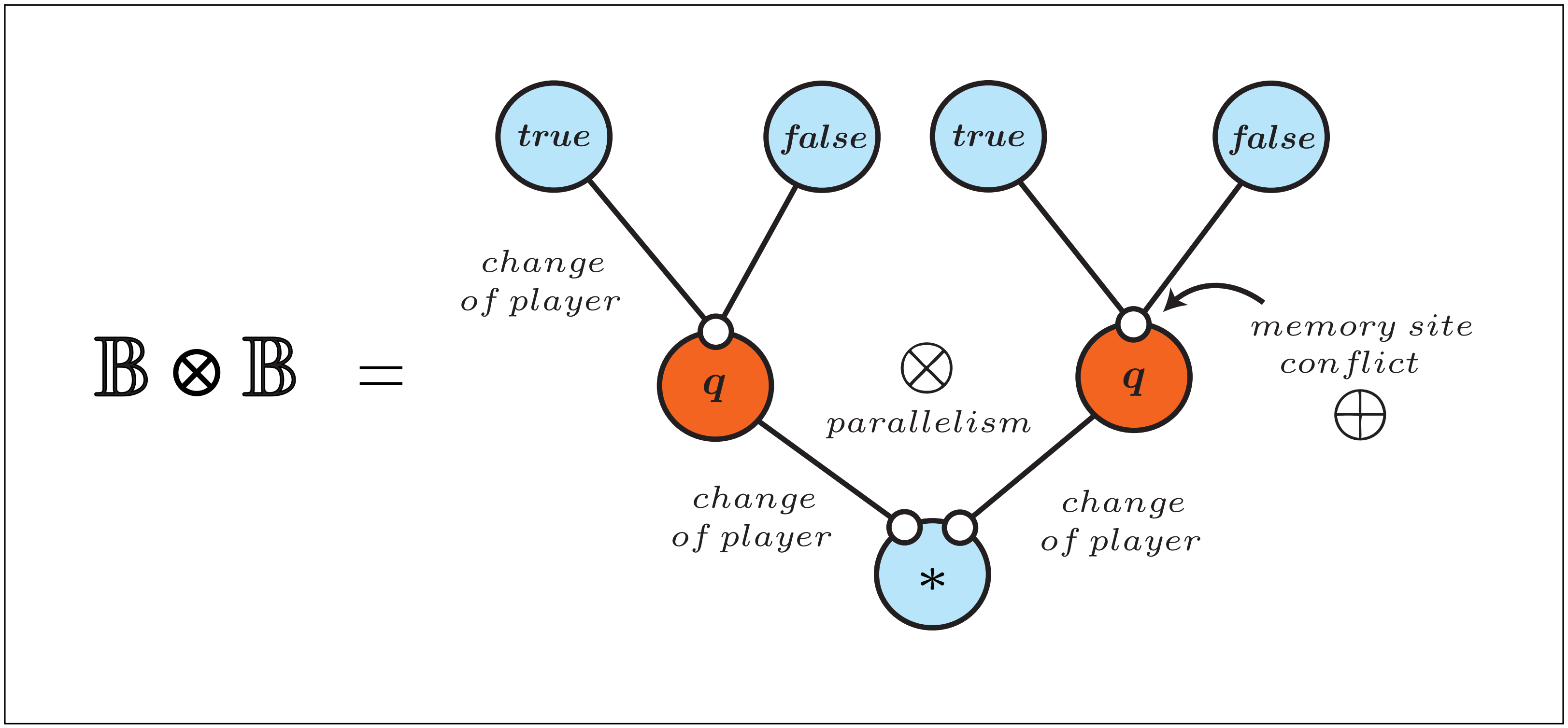
$$\begin{array}{lcl} \alpha \triangleright v & \Rightarrow & \lambda(\alpha) \; = \; \lambda(v) \\ v \triangleright \alpha & \Rightarrow & \lambda(\alpha) \; = \; -\lambda(v). \end{array}$$

Finally, one requires that the root $*$ of the tree is a value of polarity $+1$.

Dialogue games



Dialogue games



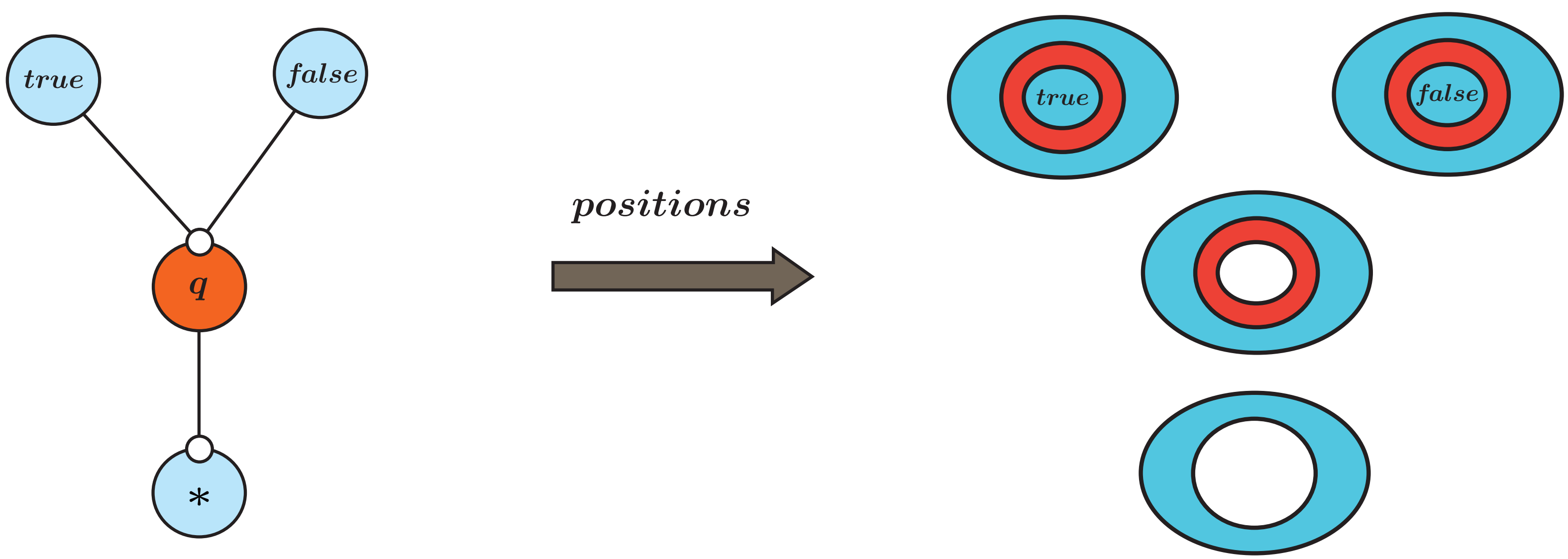
Positions of a dialogue game

The positional nature of logical proofs

Positions of a dialogue game

$$
\begin{array}{rcl}
\mathbf{Pos}\,(0) & = & \emptyset \\
\mathbf{Pos}\,(1) & = & \{\,*\,\} \\
\mathbf{Pos}\,(A \oplus B) & = & \mathbf{Pos}\,(A) \;+\; \mathbf{Pos}\,(B) \\
\mathbf{Pos}\,(A \otimes B) & = & \mathbf{Pos}\,(A) \;\times\; \mathbf{Pos}\,(B) \\
\mathbf{Pos}\,(\neg\, A) & = & \mathbf{Pos}\,(A) \;+\; \{\,*\,\}
\end{array}
$$

Positions of a dialogue game



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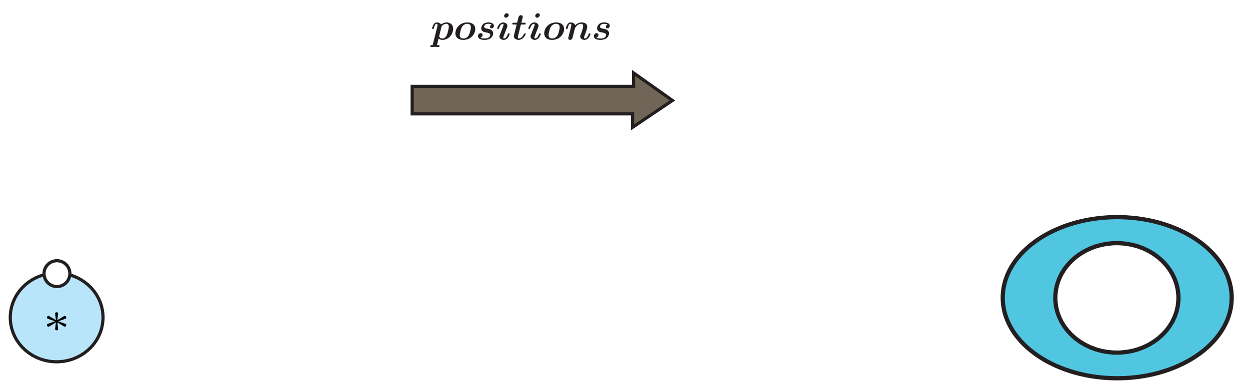

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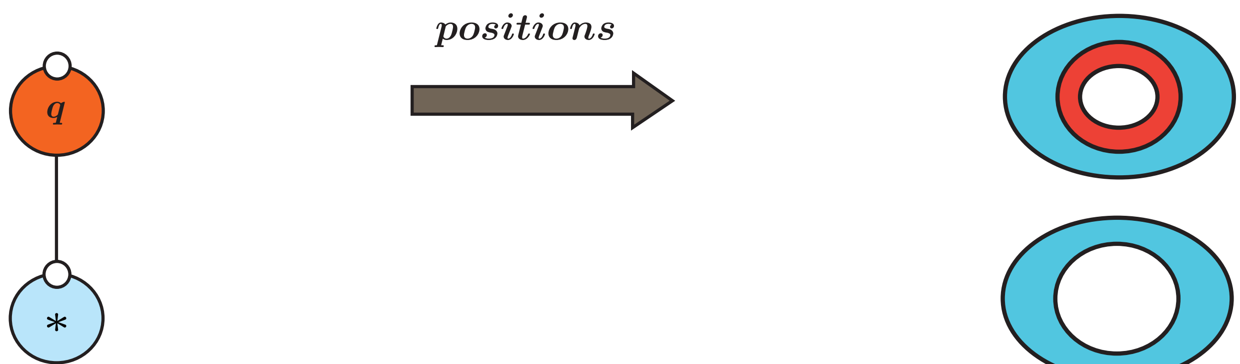

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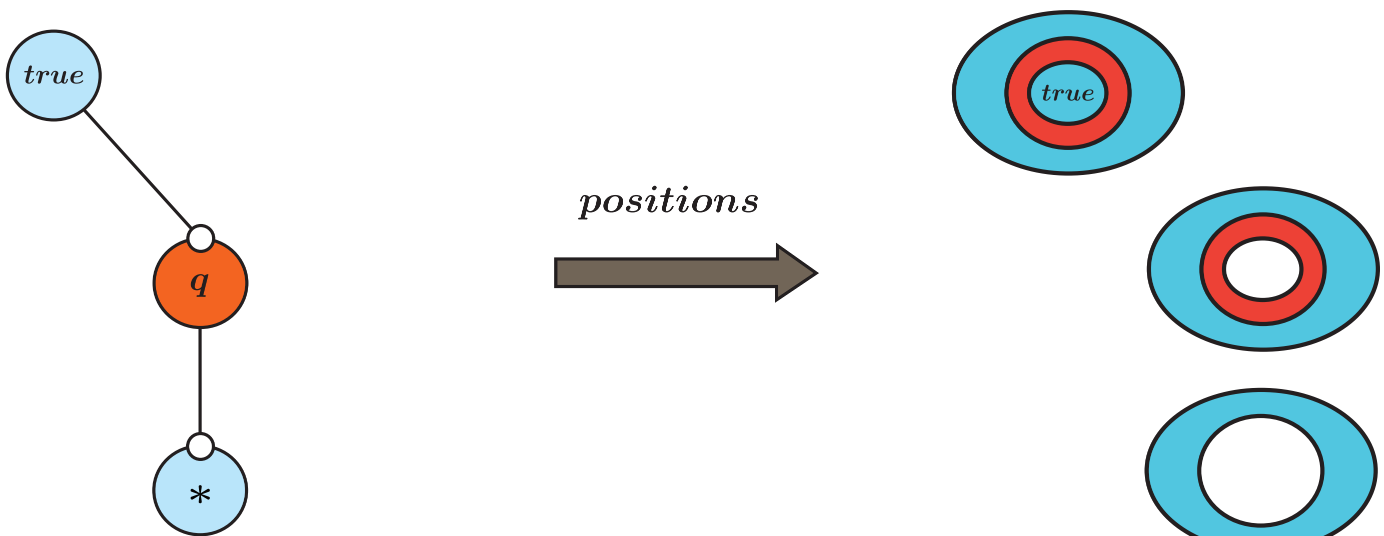

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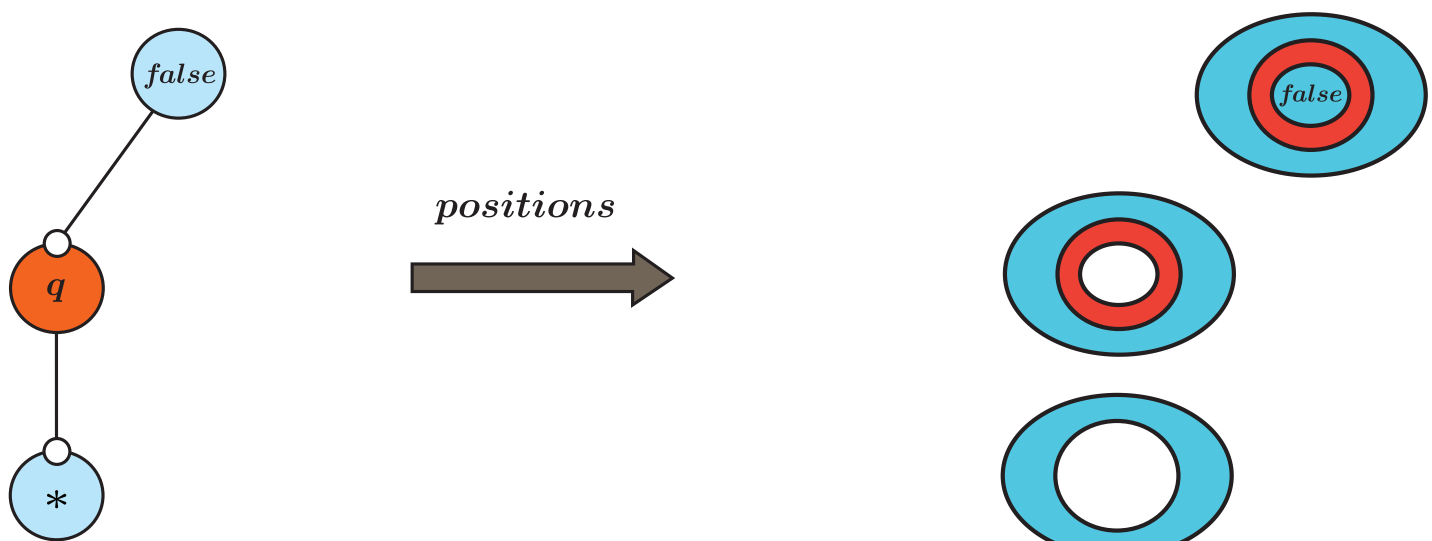

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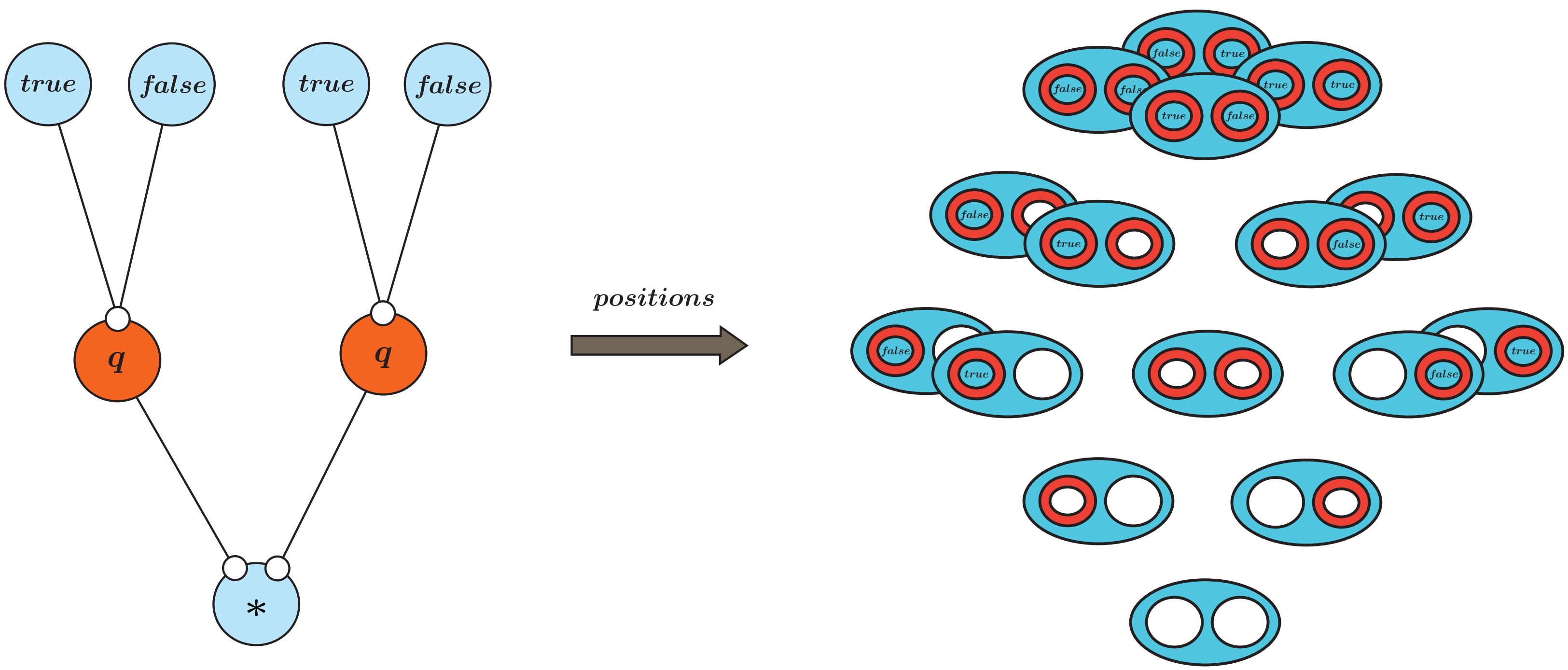

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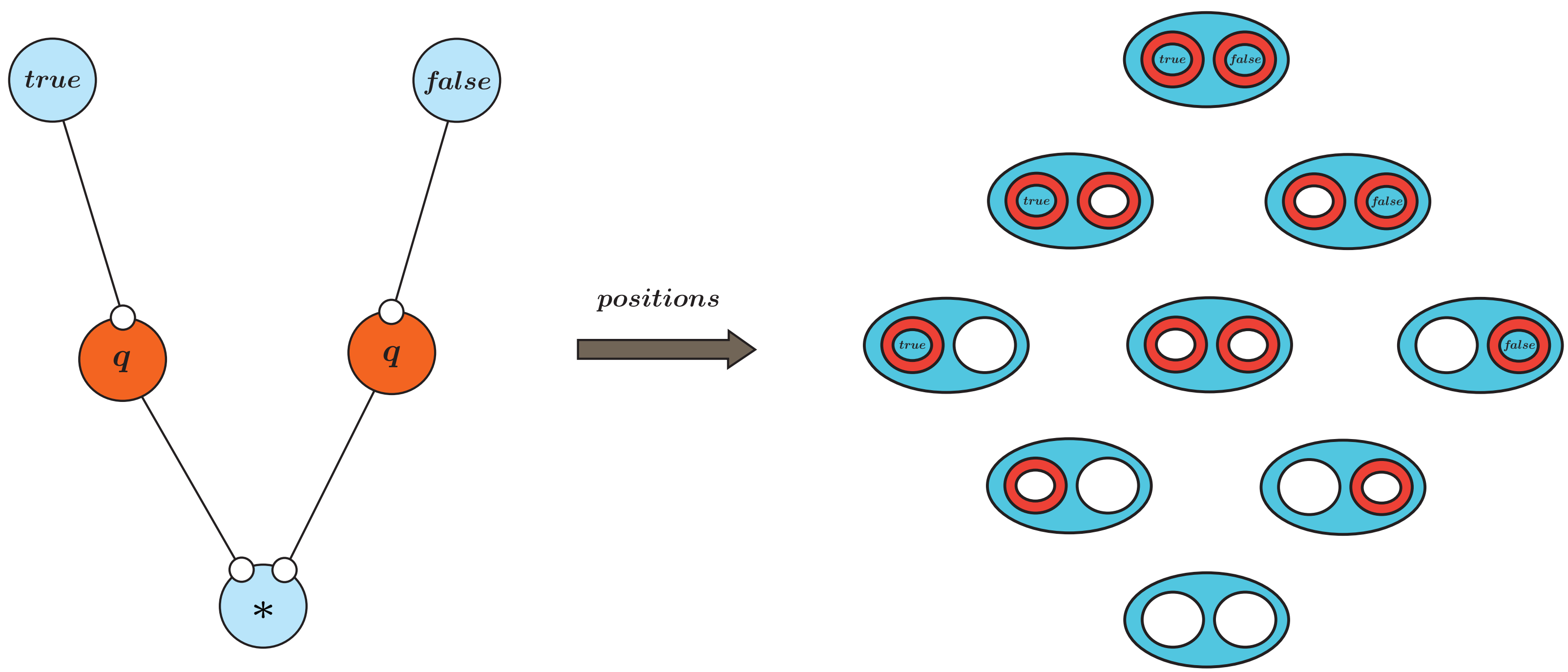

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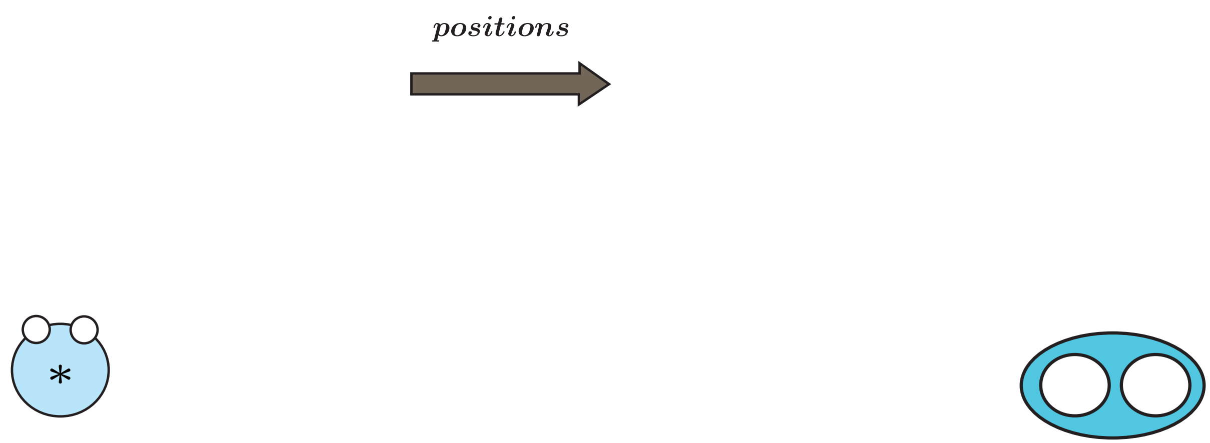

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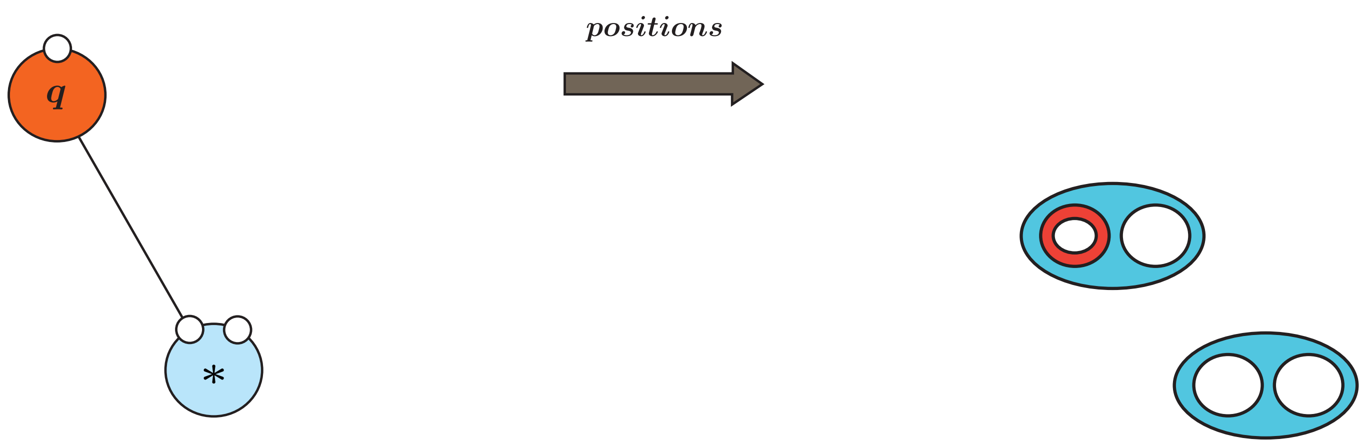

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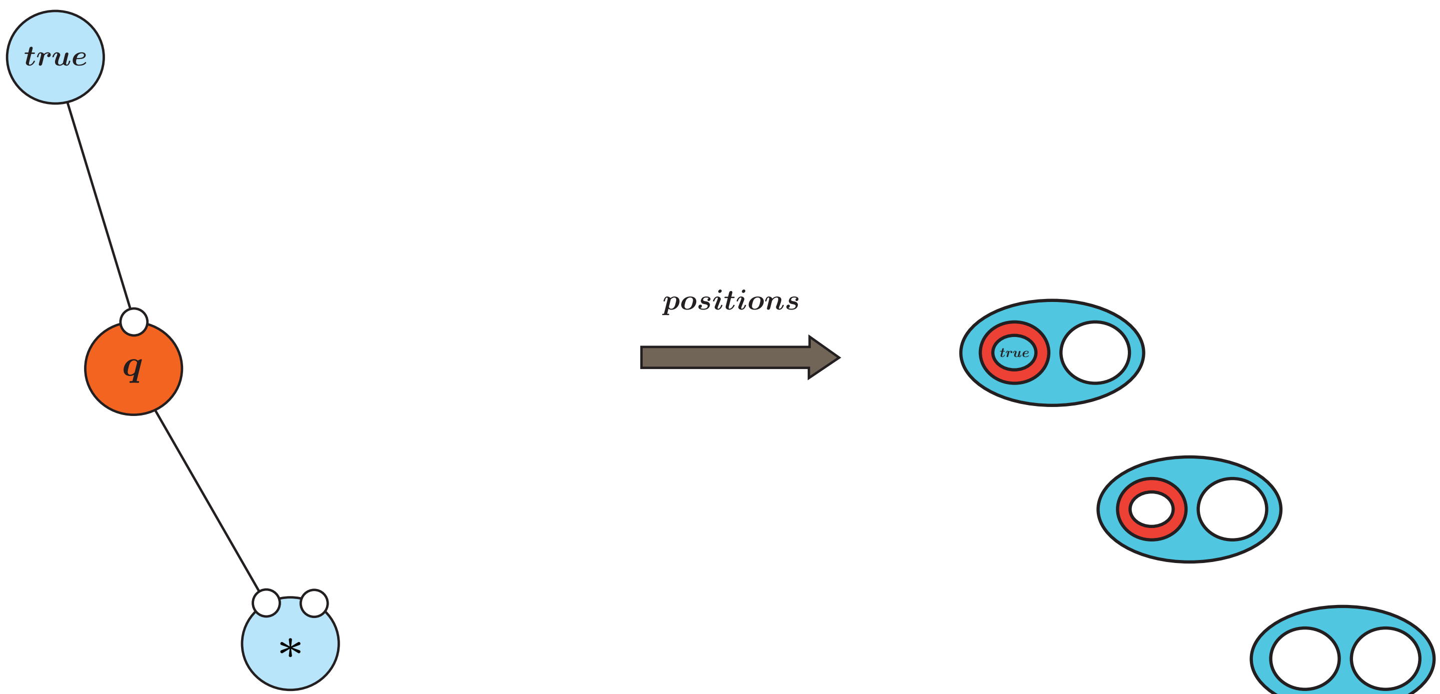

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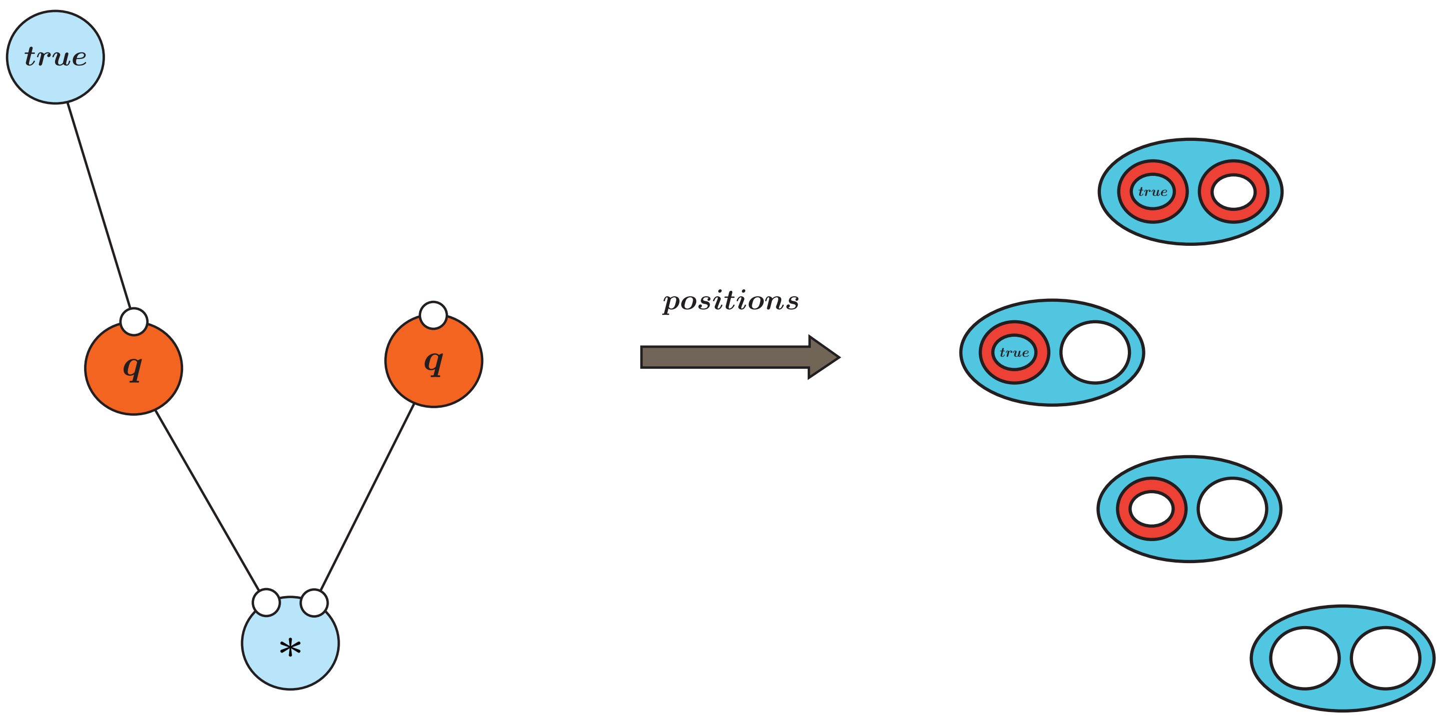

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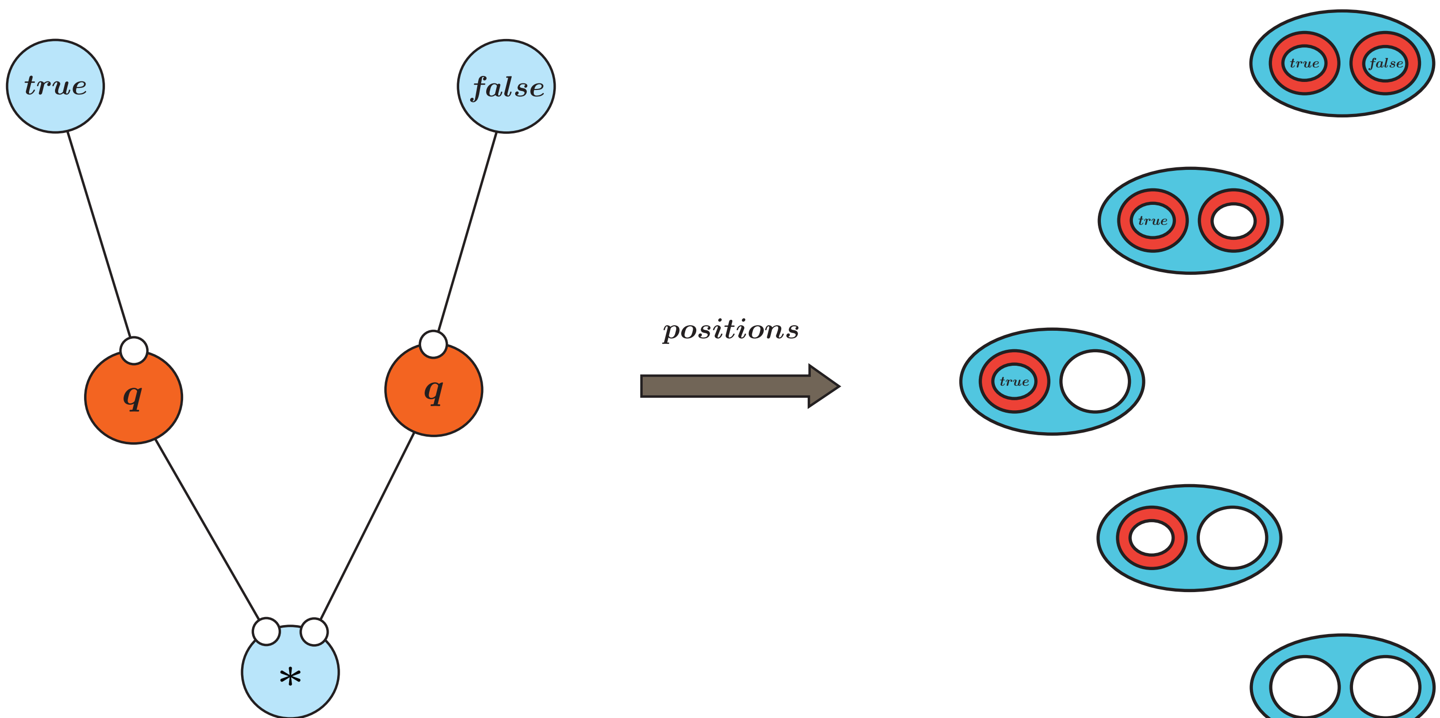

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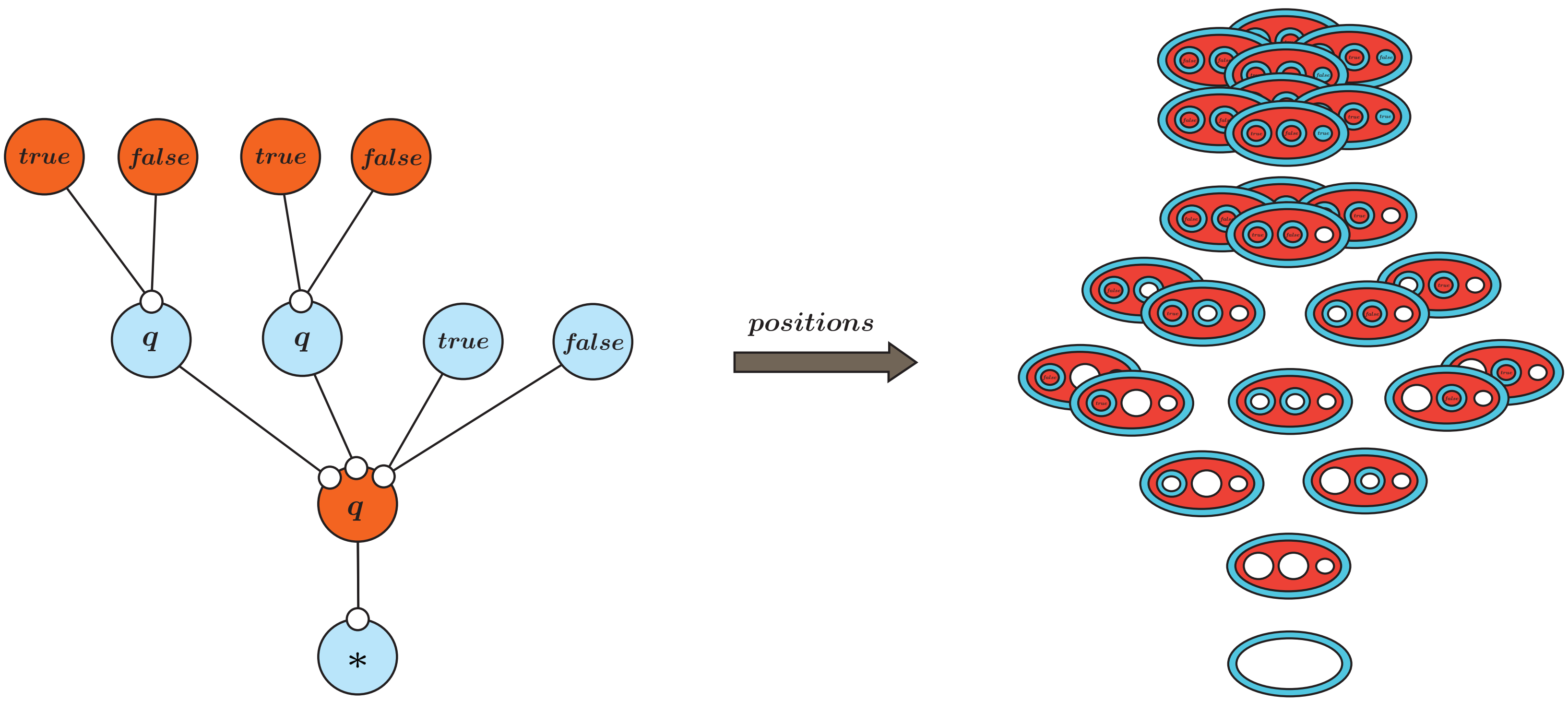
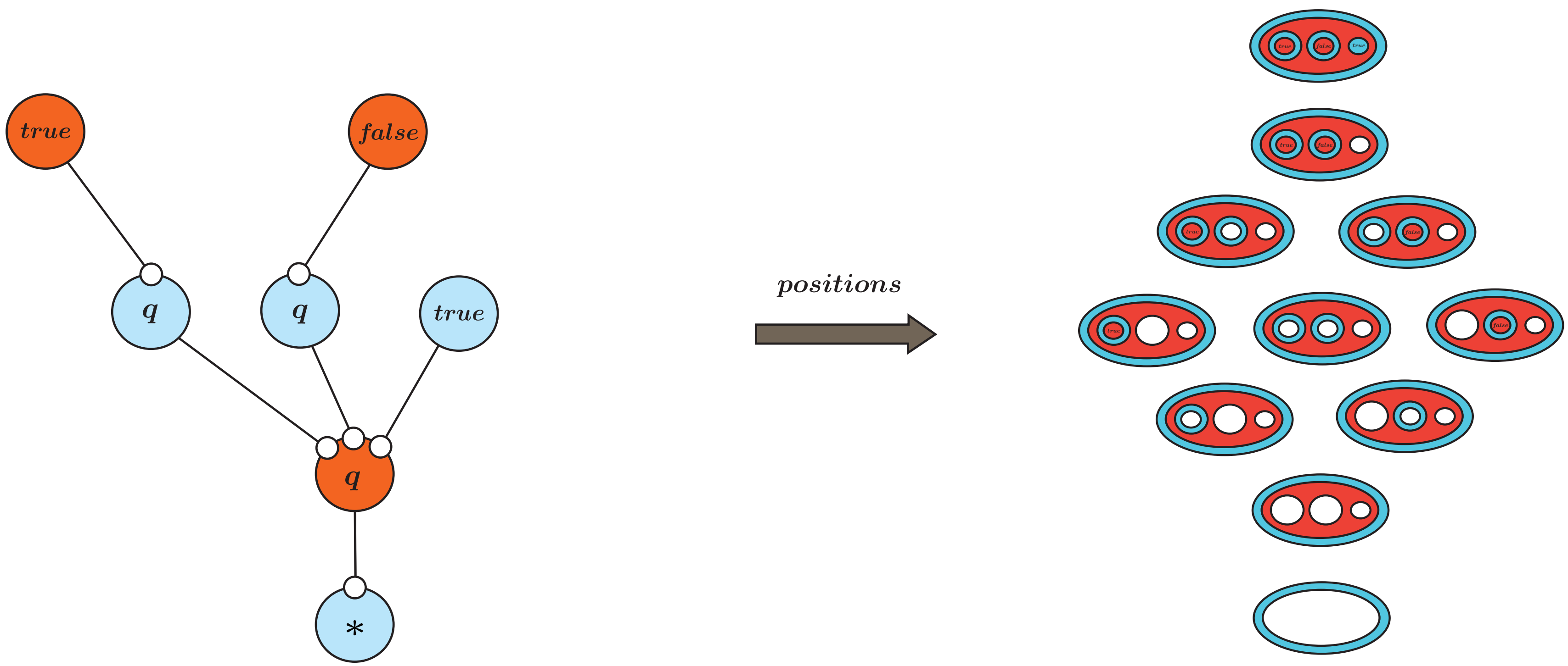


$$\mathbb{B} \otimes \mathbb{B} \multimap \mathbb{B}$$

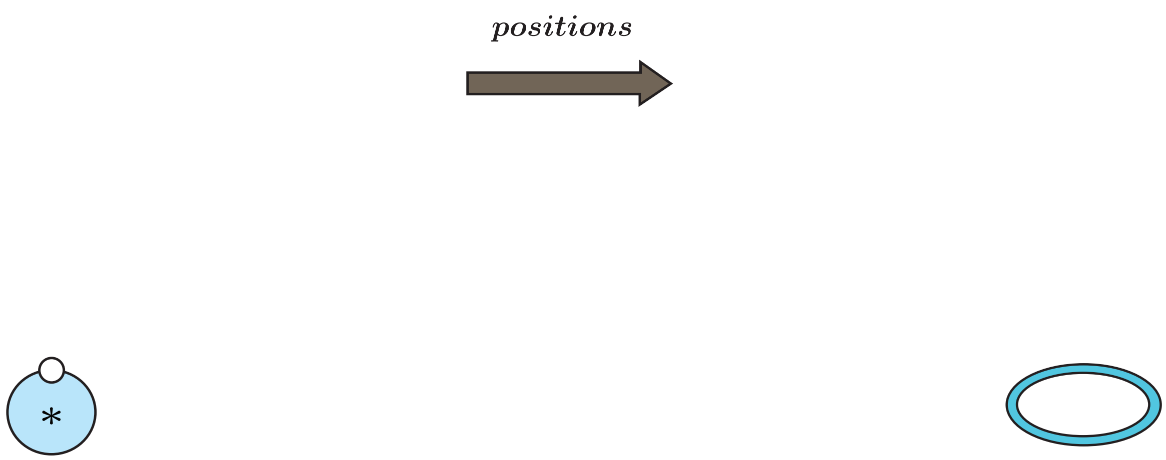
Positions of a dialogue game



The left-to-right strategy

$\neg\neg$ `bool`	$\times$	$\neg\neg$ `bool`	$\Rightarrow$	$\neg\neg$ `bool`
				`question`
`question`				
`true`				
		`question`		
		`false`		
				`true`

Positions of a dialogue game



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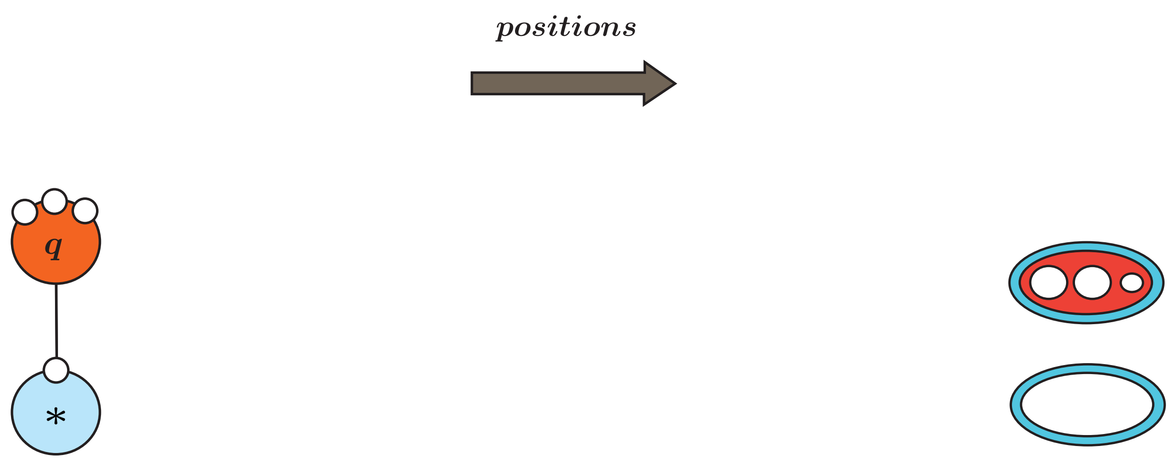

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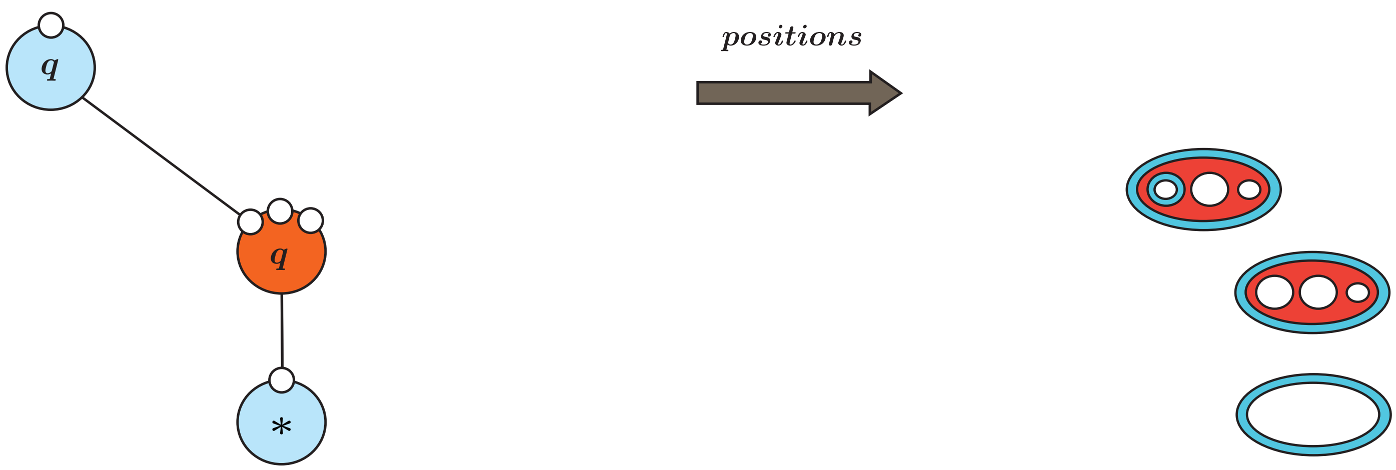

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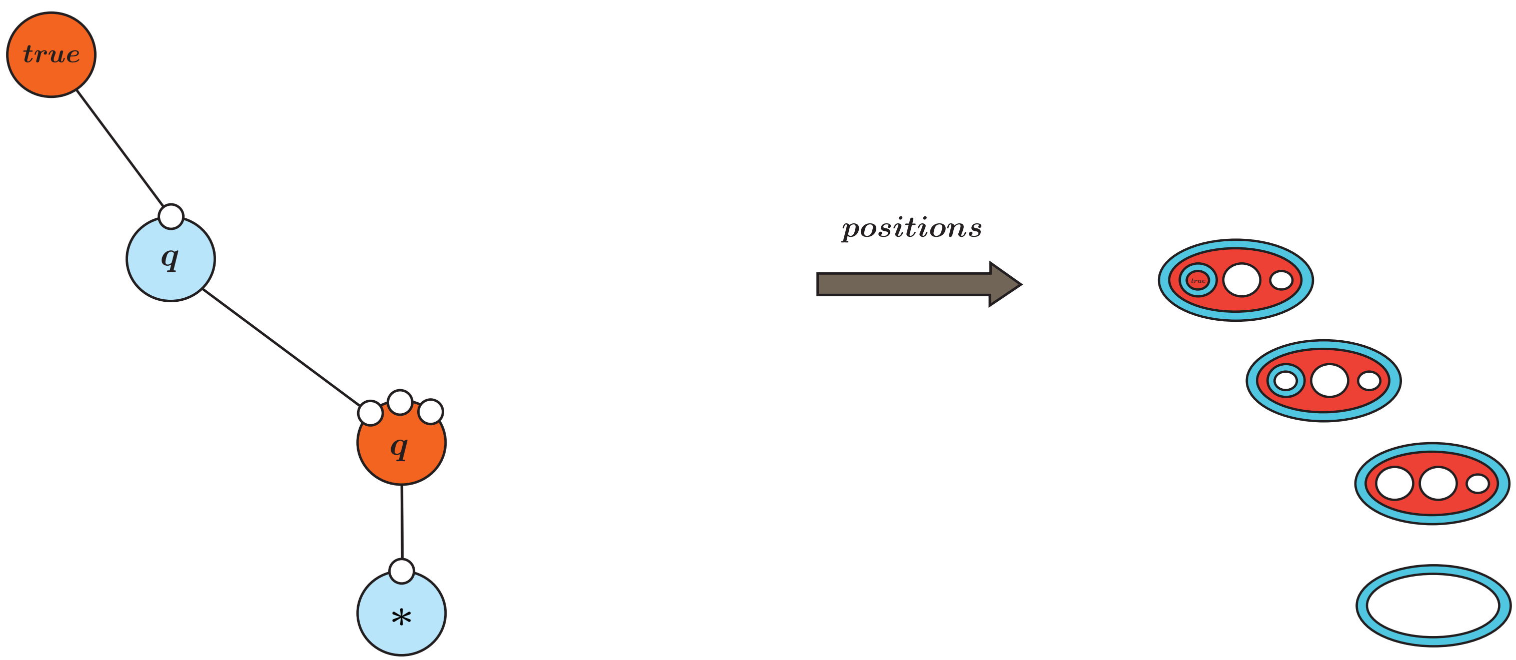

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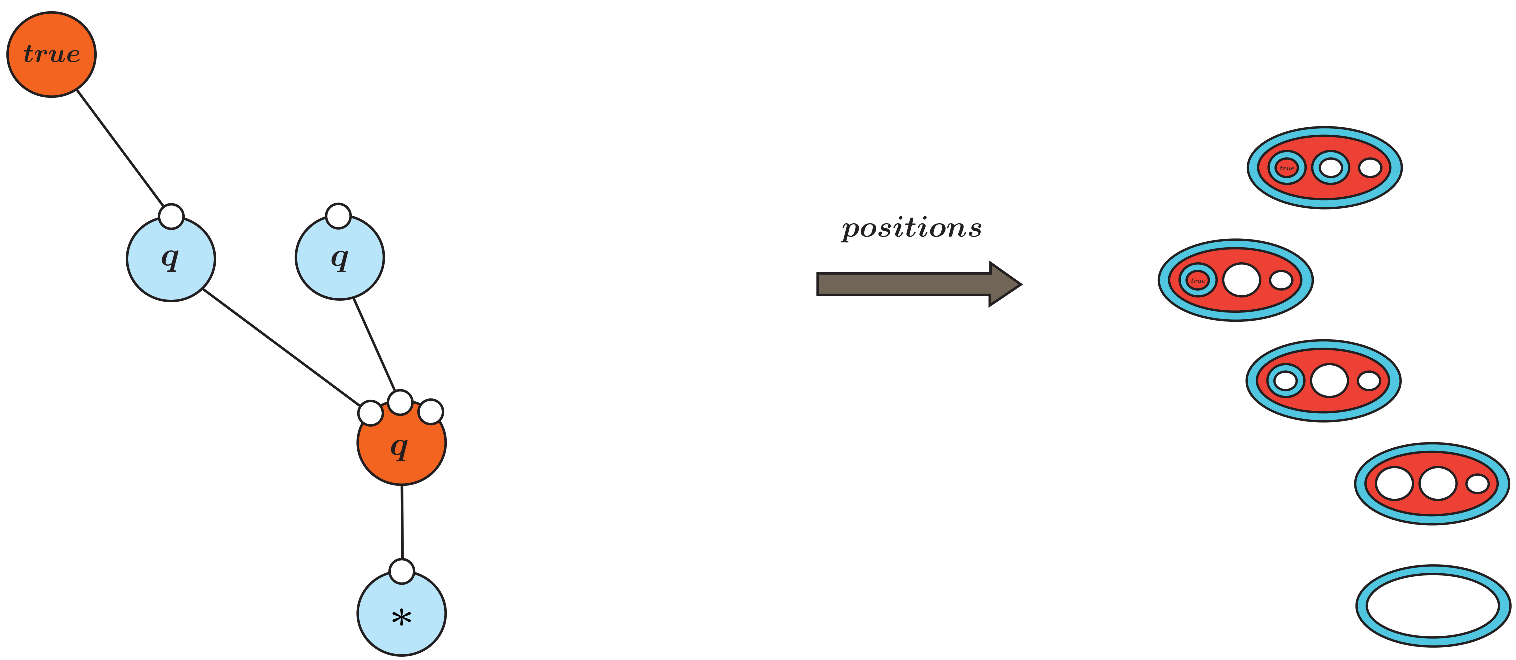

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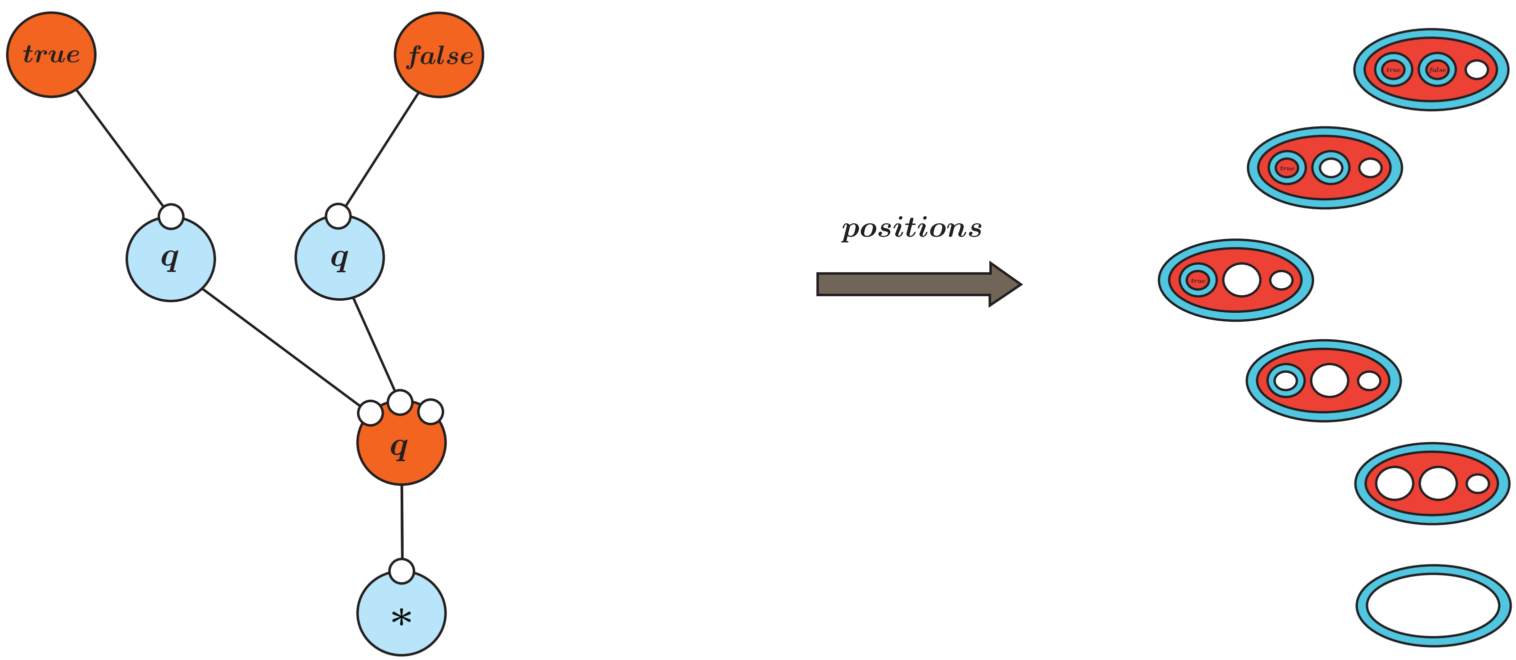

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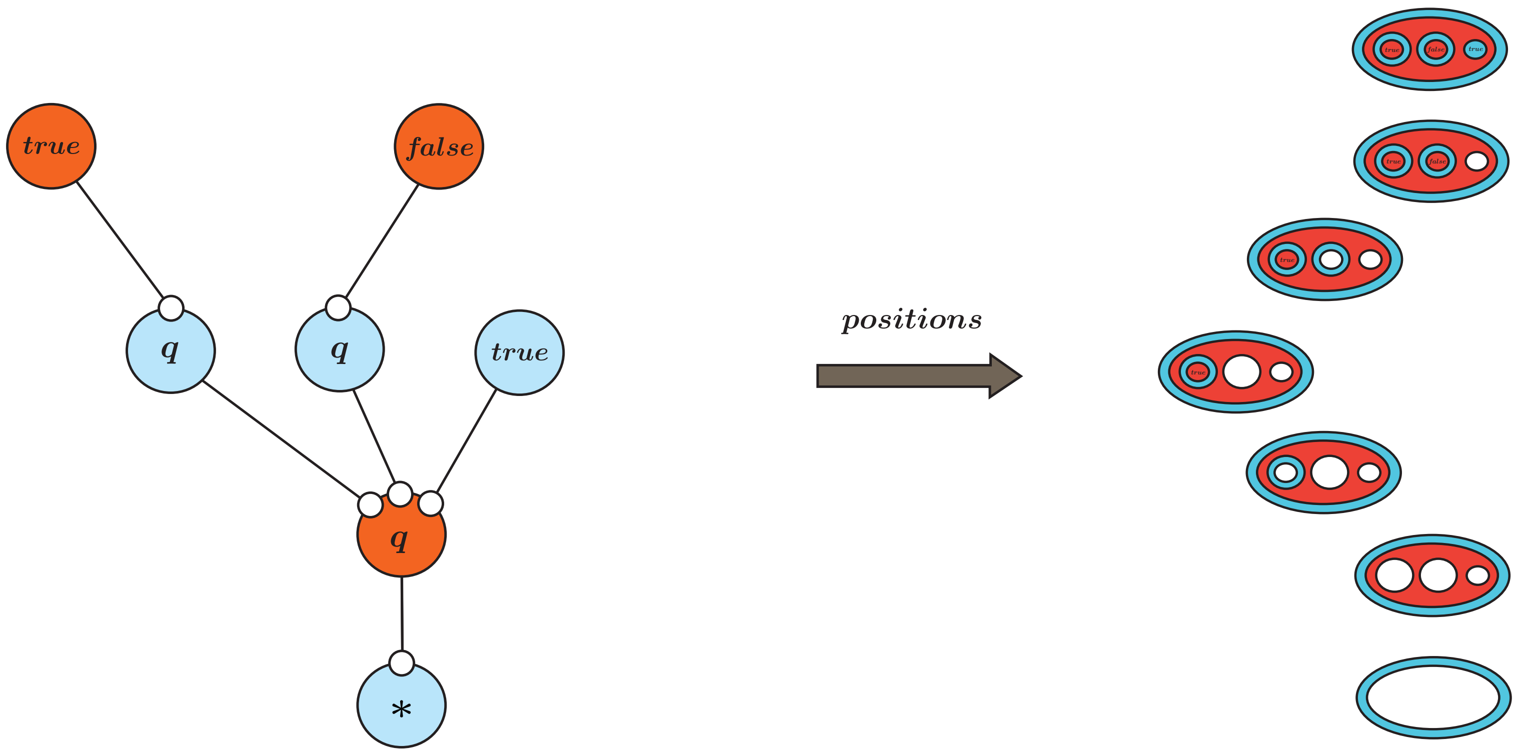
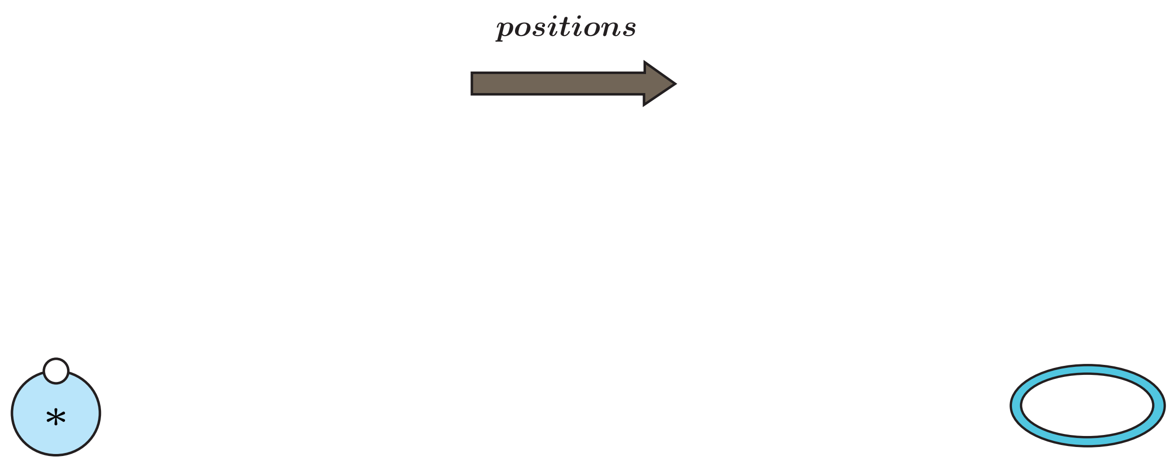


The right-to-left strategy

$\neg\neg$ bool	$\times$	$\neg\neg$ bool	$\Rightarrow$	$\neg\neg$ bool
				question
		question		
		false		
question				
true				
				true

Positions of a dialogue game



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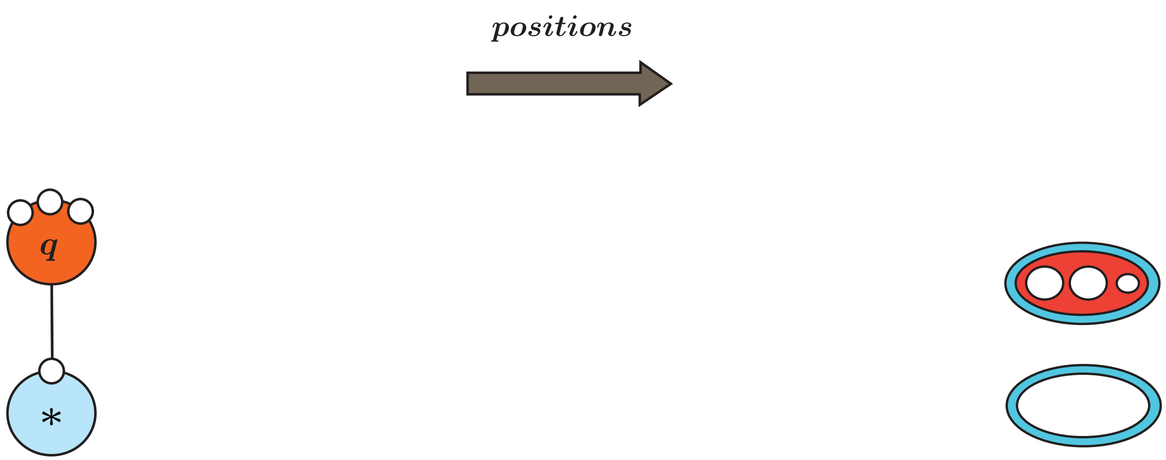

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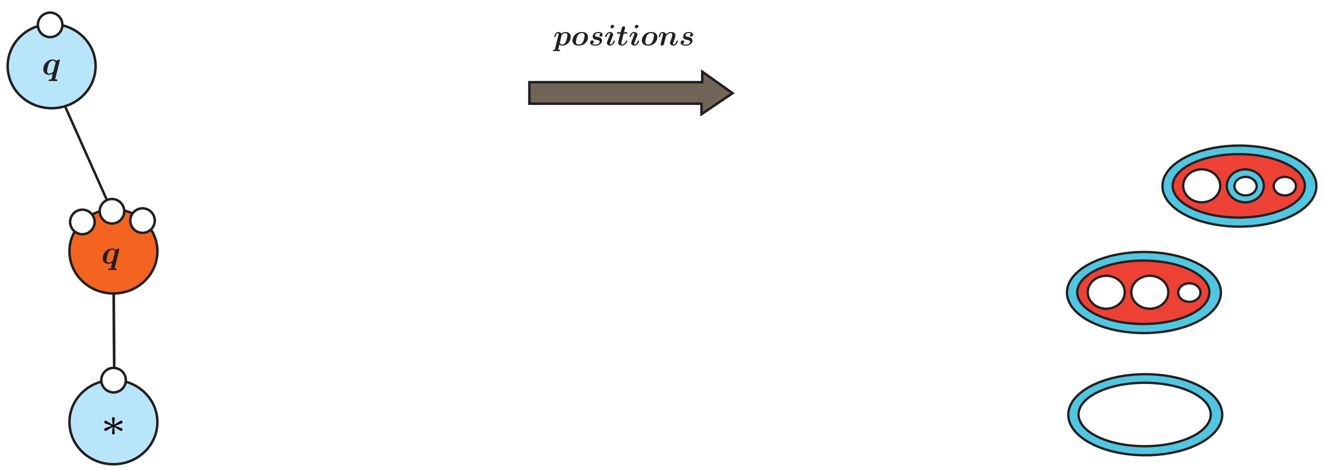

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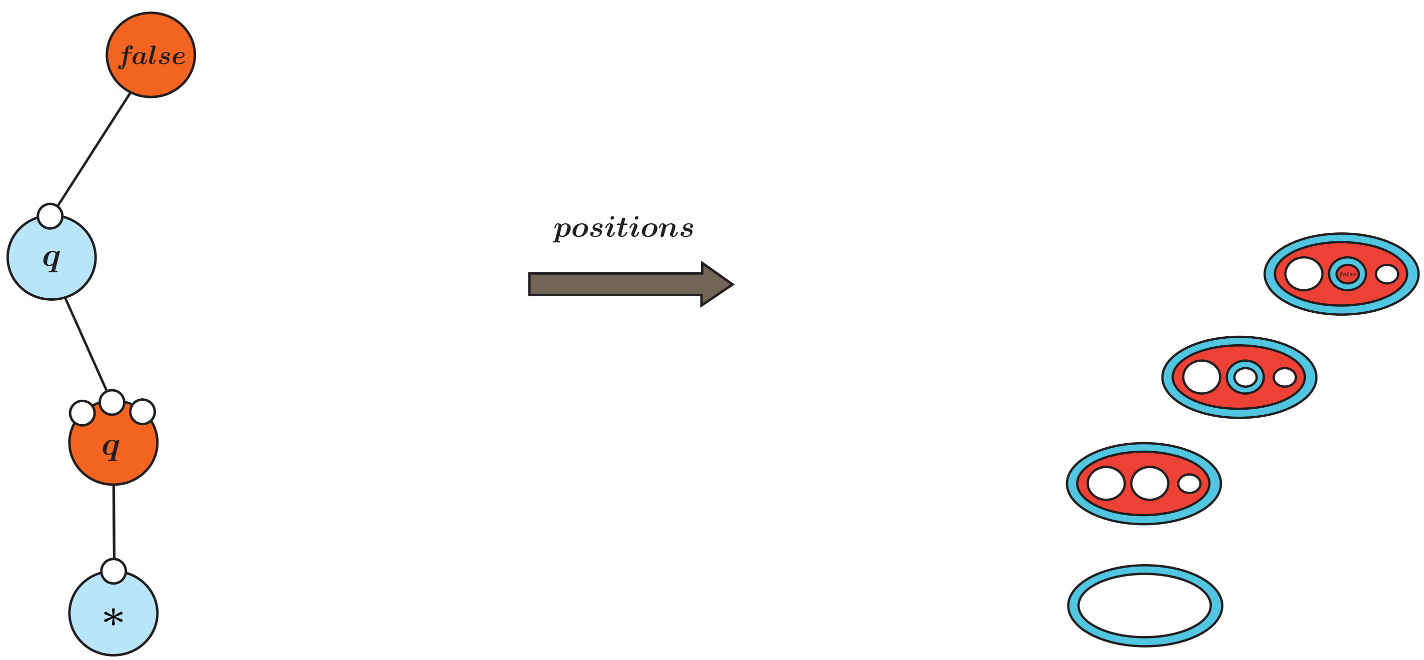

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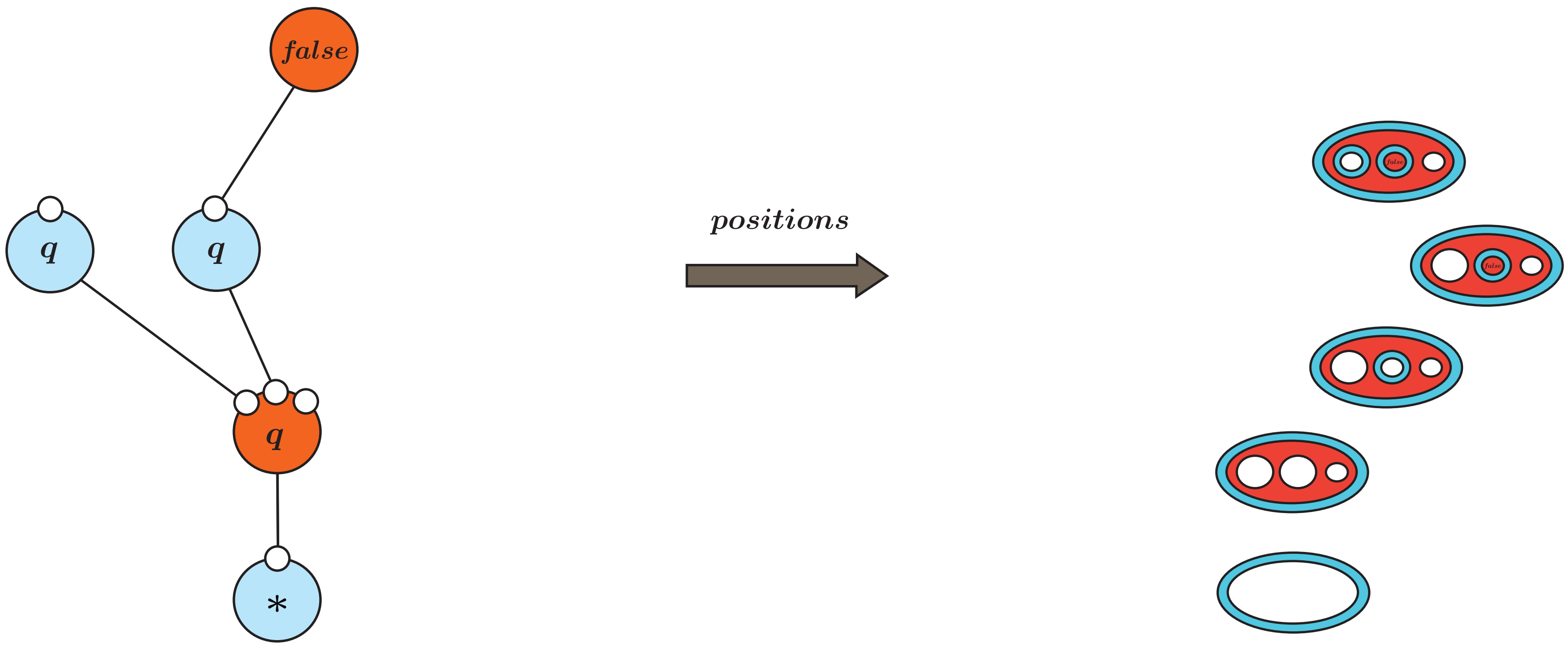
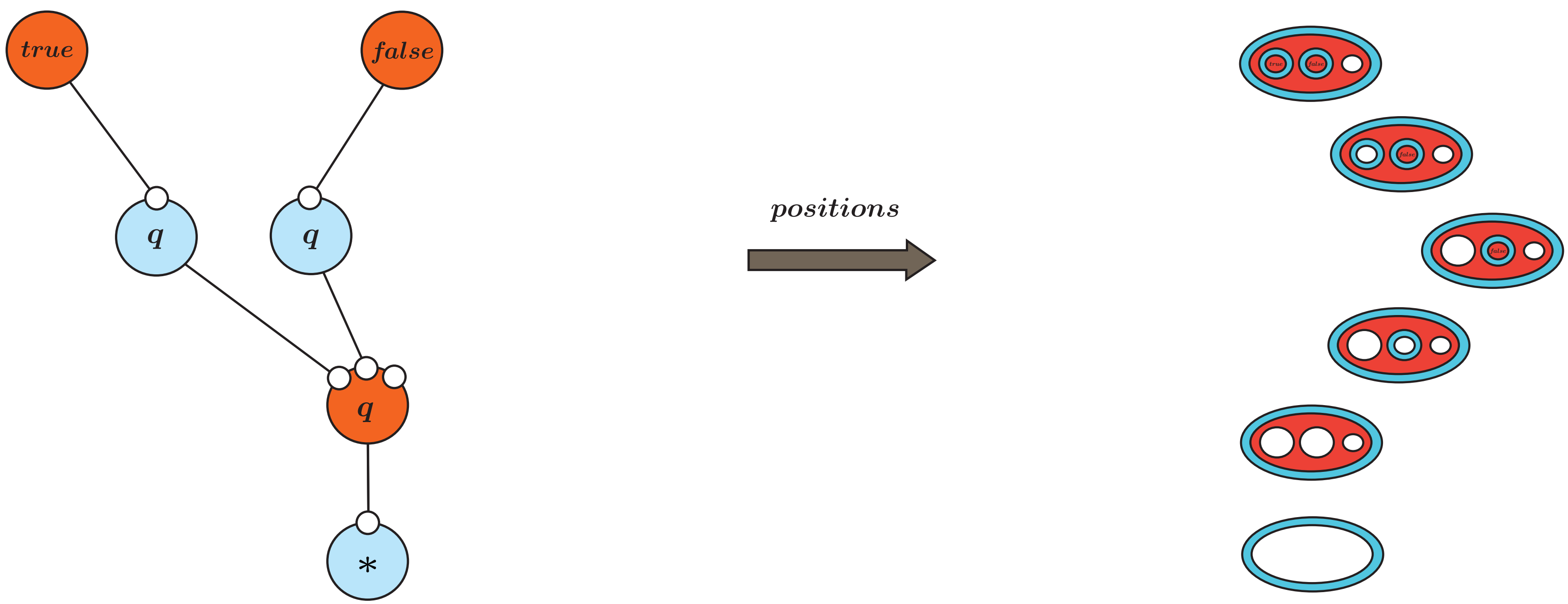
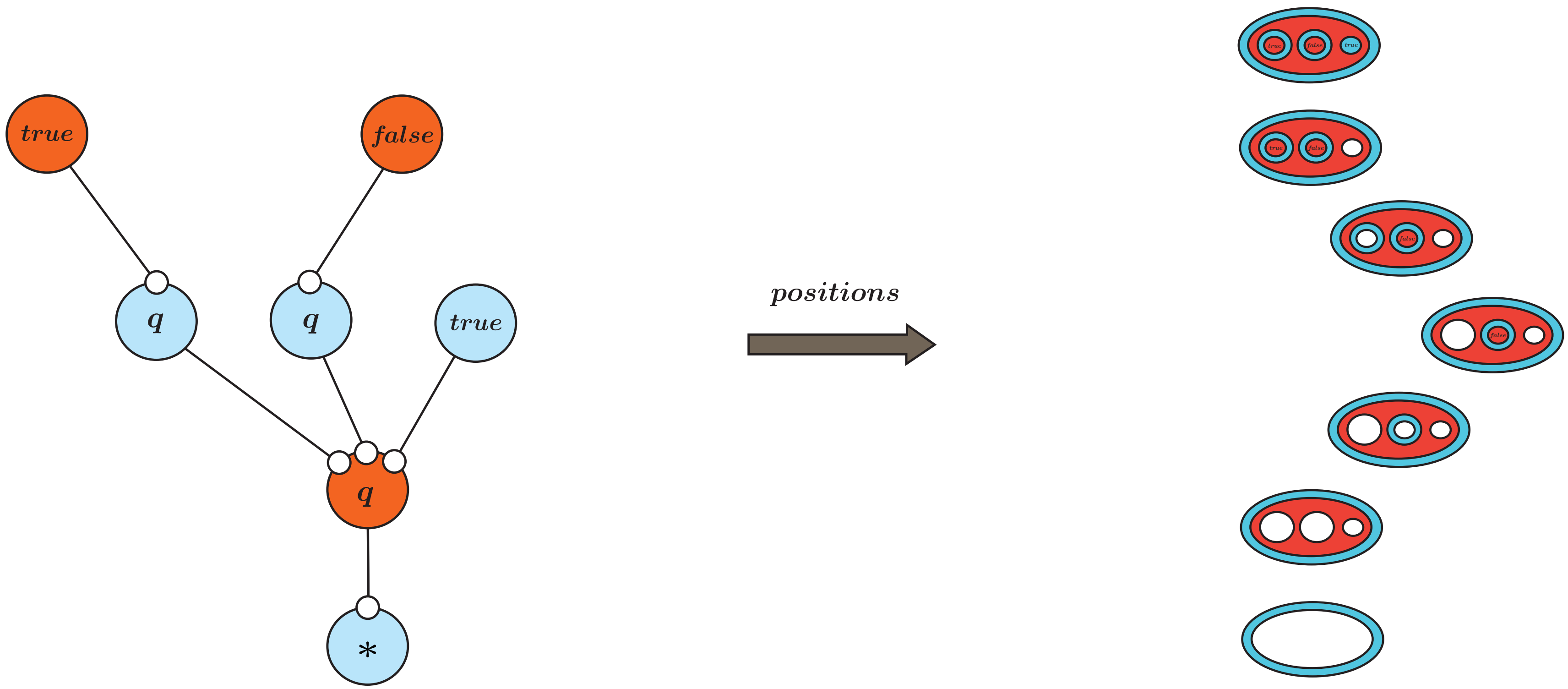


Positions of a dialogue game



Positions of a dialogue game



How to turn (functorially) a dialogue game into a hypercoherence space

Putting the free dialogue category at work !

Positions of a dialogue game

$$
\begin{aligned}
\mathbf{Pos}\,(0) &= \emptyset \\
\mathbf{Pos}\,(1) &= \{\ast\} \\
\mathbf{Pos}\,(A \oplus B) &= \mathbf{Pos}\,(A) + \mathbf{Pos}\,(B) \\
\mathbf{Pos}\,(A \otimes B) &= \mathbf{Pos}\,(A) \times \mathbf{Pos}\,(B) \\
\mathbf{Pos}\,(\neg A) &= \mathbf{Pos}\,(A) + \{\ast\}
\end{aligned}
$$

Suspension modality

Definition. A suspension modality

$$\Box \quad : \quad \mathcal{C} \quad \longrightarrow \quad \mathcal{C}$$

on a symmetric monoidal category $\mathcal{C}$ is defined as a

symmetric monoidal comonad

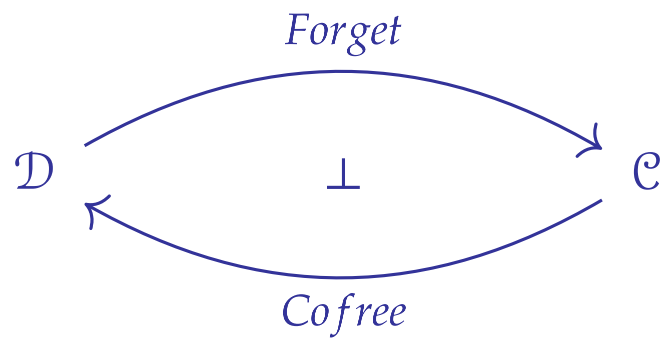
that is, a functor equipped with maps

ε_A	:	$\Box A$	$\longrightarrow$	A	counit
δ_A	:	$\Box A$	$\longrightarrow$	$\Box \Box A$	comultiplication
$m_{A,B}$	:	$\Box A \otimes \Box B$	$\longrightarrow$	$\Box (A \otimes B)$	lax monoidal
m_1	:	1	$\longrightarrow$	$\Box 1$	lax monoidal

satisfying a series of coherence properties.

A Gödel translation $L\otimes \rightsquigarrow LL + \Box$

Fact. Every suspension modality $\Box$ induces an adjunction



between the category $\mathcal{C}$ and its category $\mathcal{D}$ of $\Box$-coalgebras.

Key observation.

When $\mathcal{C}$ is $*$-autonomous, $\mathcal{D}$ is a dialogue category with negation

$$\neg A \quad = \quad \mathit{Cofree}\,(\,(\mathit{Forget}\,A)^*\,)$$

A Gödel translation $L\otimes \rightsquigarrow LL + \Box$

Corollary.

Every $*$-autonomous category $\mathcal{C}$ equipped with a suspension modality $\Box$ induces a functor

$$\mathcal{F} \xrightarrow{[-]} \mathcal{D} \xrightarrow{Forget} \mathcal{C}$$

which satisfies the equations

$$\begin{array}{rcl}
[0] & = & 0 \\
[1] & = & 1 \\
[A \oplus B] & = & [A] \oplus [B] \\
[A \otimes B] & = & [A] \otimes [B] \\
[\neg A] & = & \Box\,(\,[A]^*\,)
\end{array}$$

Four functorial interpretations

$\mathcal{F}$	$\longrightarrow$	$\mathbf{Rel}$	sets and relations
$\mathcal{F}$	$\longrightarrow$	$\mathbf{Coh}$	coherence spaces and cliques
$\mathcal{F}$	$\longrightarrow$	$\mathbf{HCoh}$	hypercoherence spaces and cliques
$\mathcal{F}$	$\longrightarrow$	$\mathbf{BiStr}$	bistructures and configurations

Corollary.

The set $\mathbf{Pos}\,(A)$ of positions of a dialogue game defines:

- ▷ a coherence space,
- ▷ a hypercoherence space,
- ▷ a bistructure.

The Gustave formula in tensorial logic

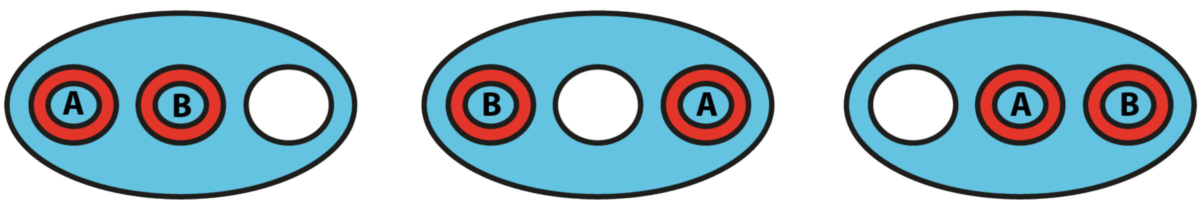
The Gustave formula is defined as

$$\mathbb{G} \quad = \quad \mathbb{B} \otimes \mathbb{B} \otimes \mathbb{B}$$

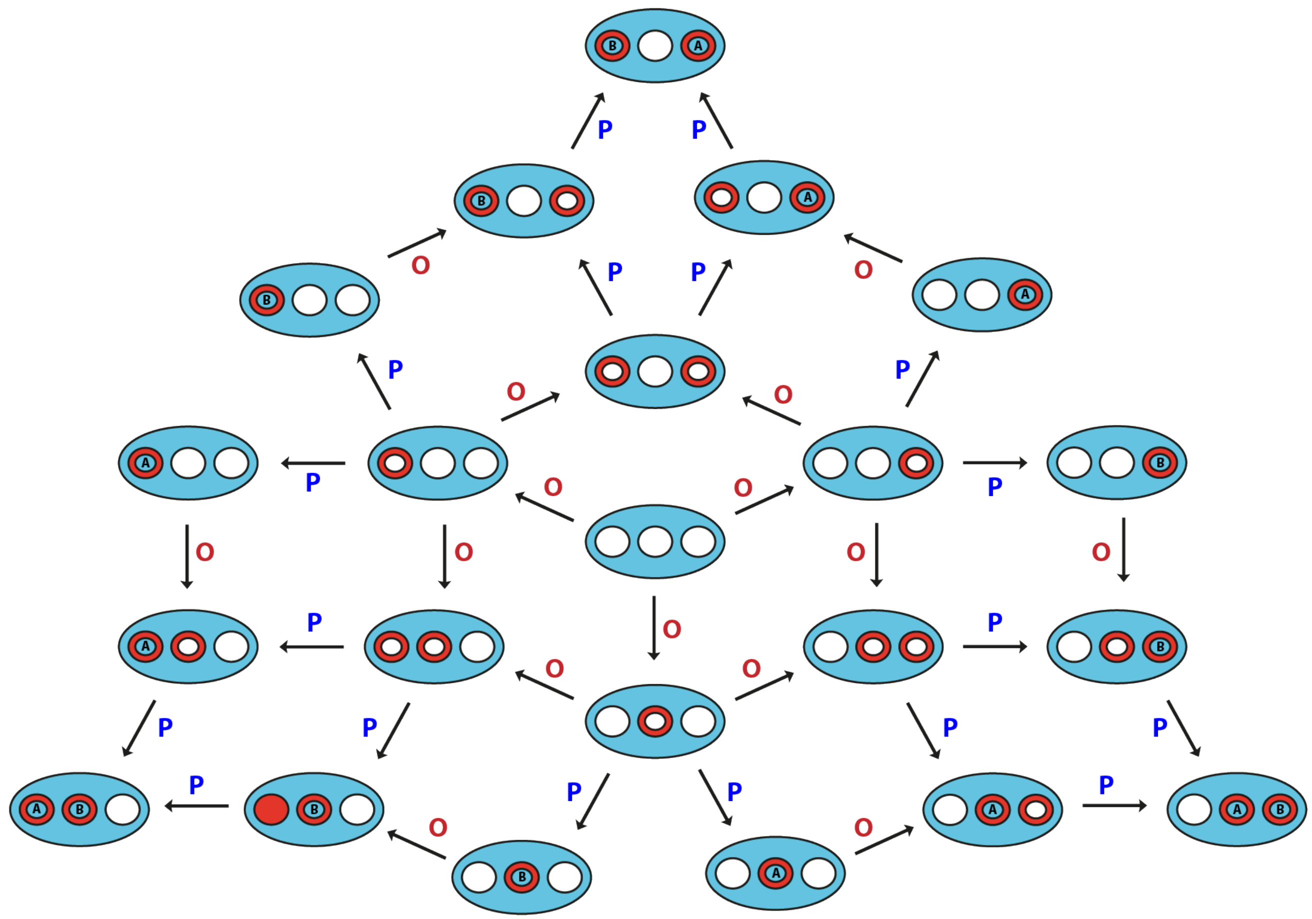
where the boolean formula is defined using a double negation:

$$\mathbb{B} \quad = \quad \neg\neg(1 \oplus 1)$$

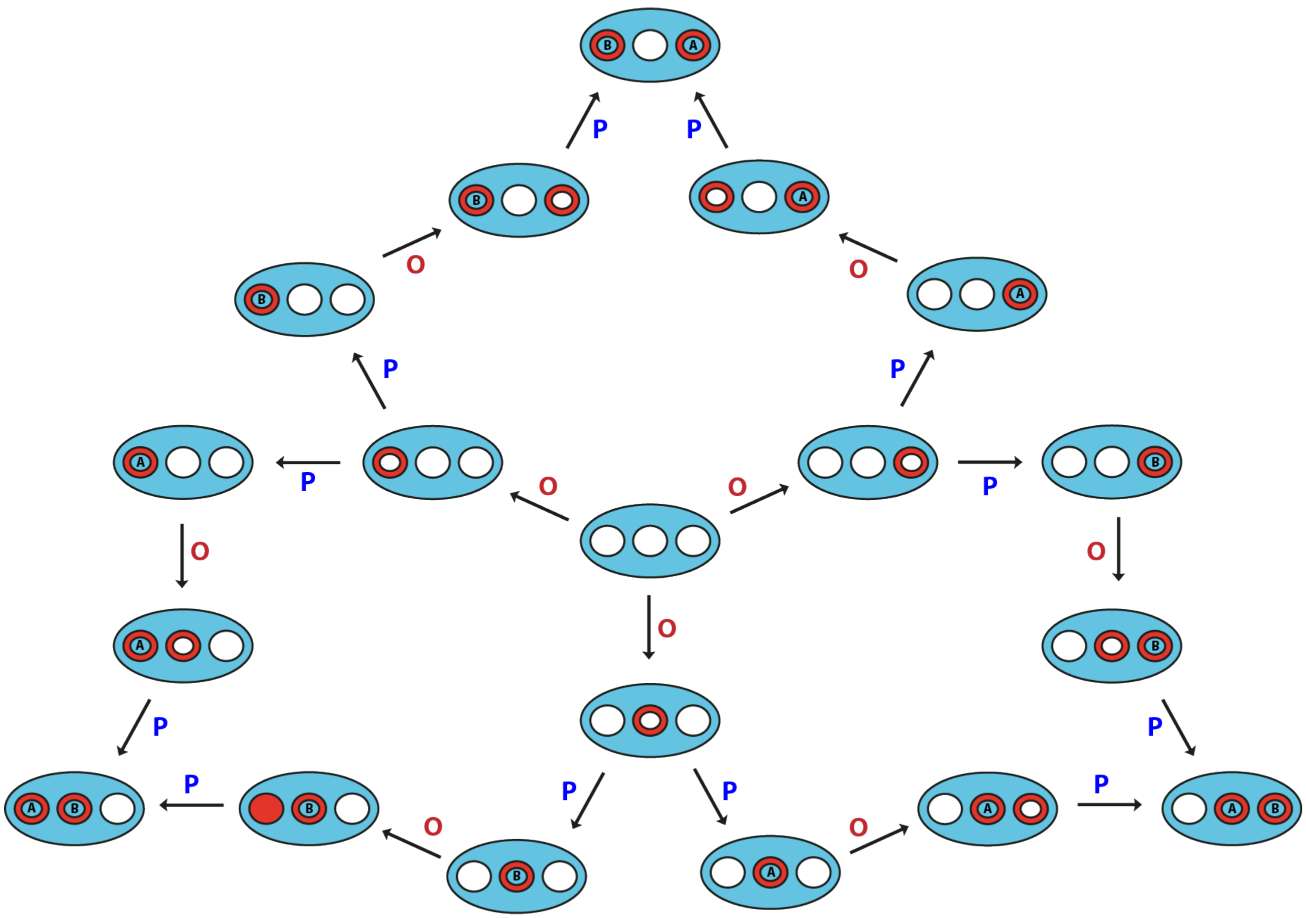
We are interested in the three positions in the dialogue game:



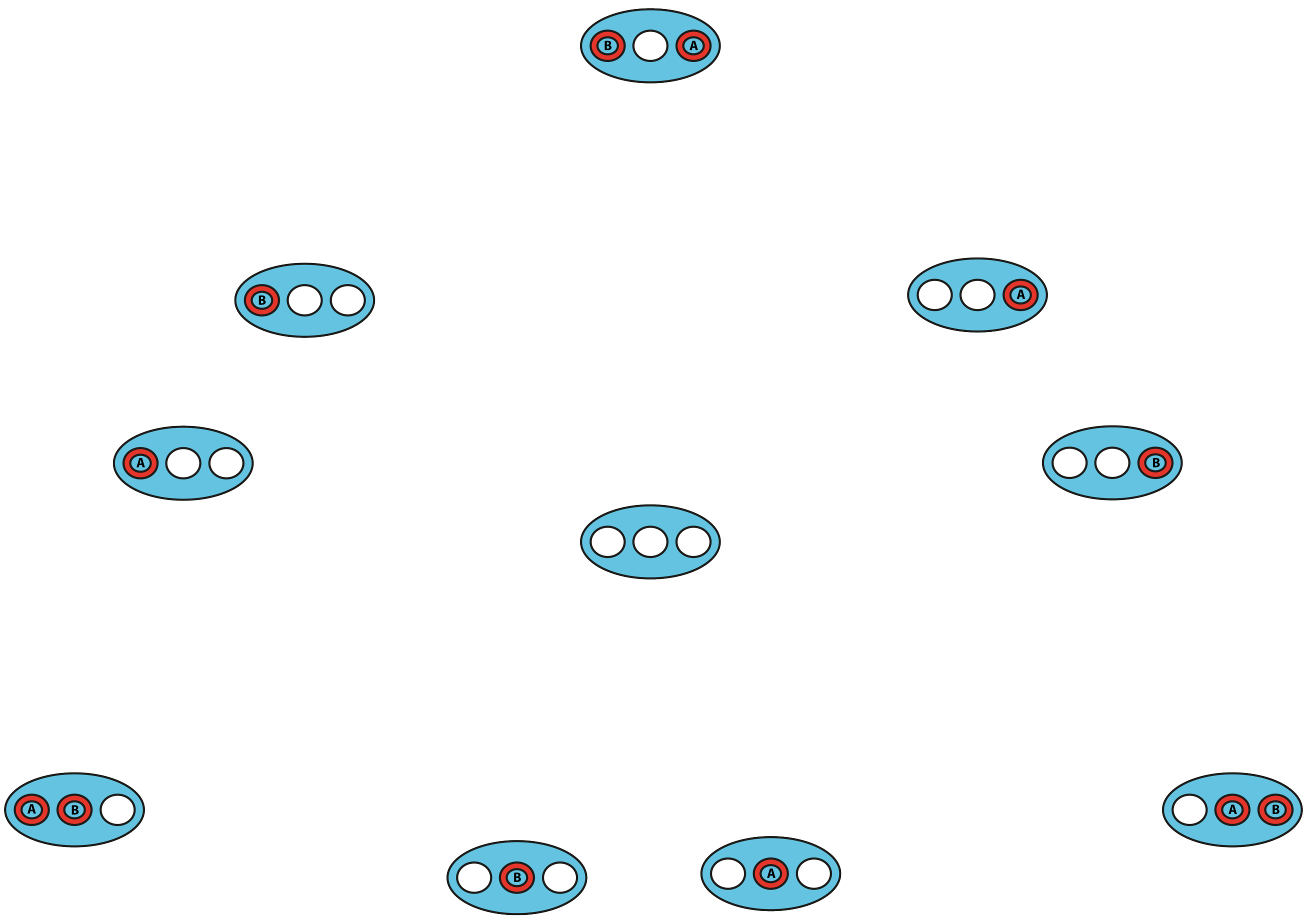
The Gustave formula in dialogue games



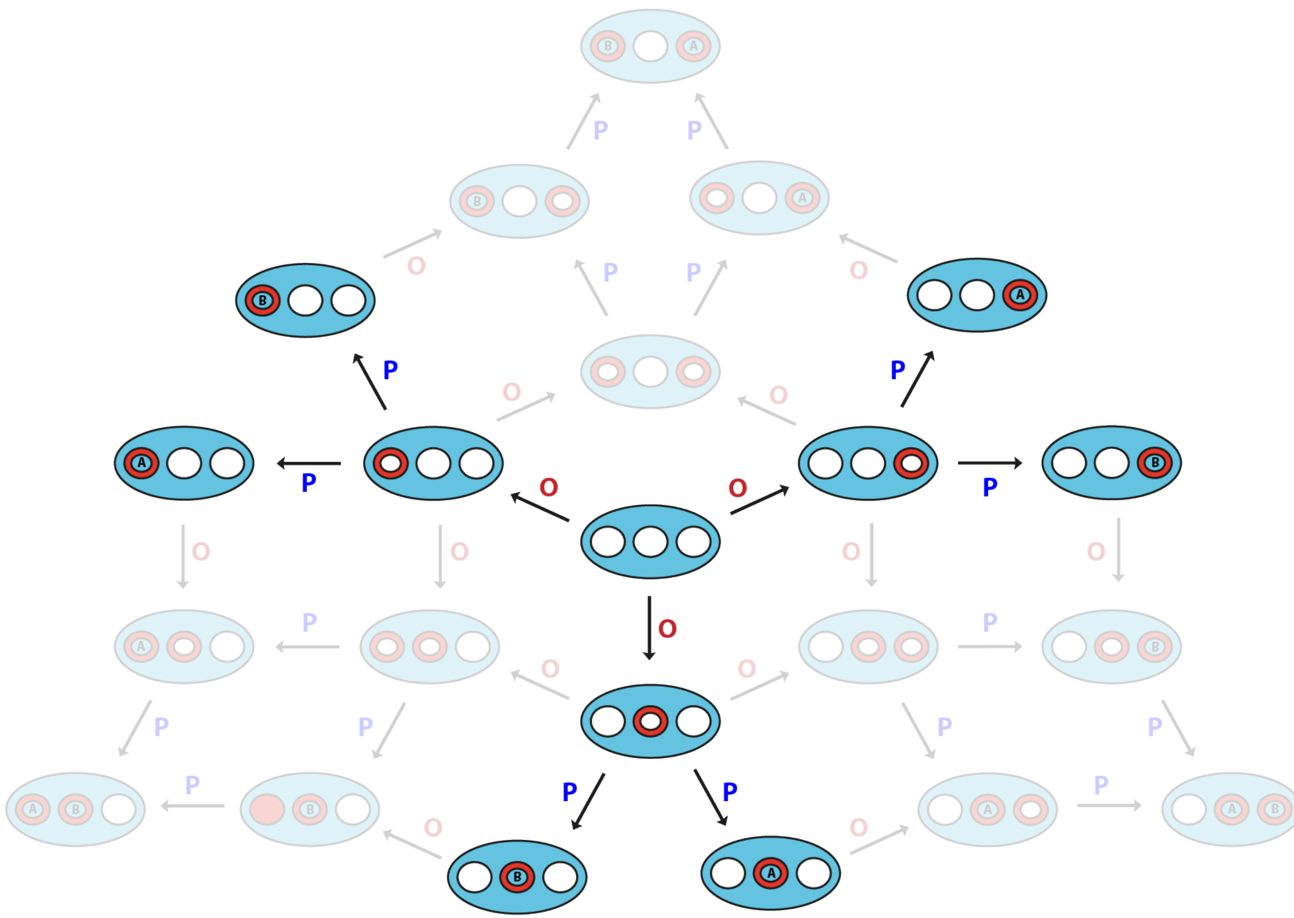
The Gustave formula in sequential data structures



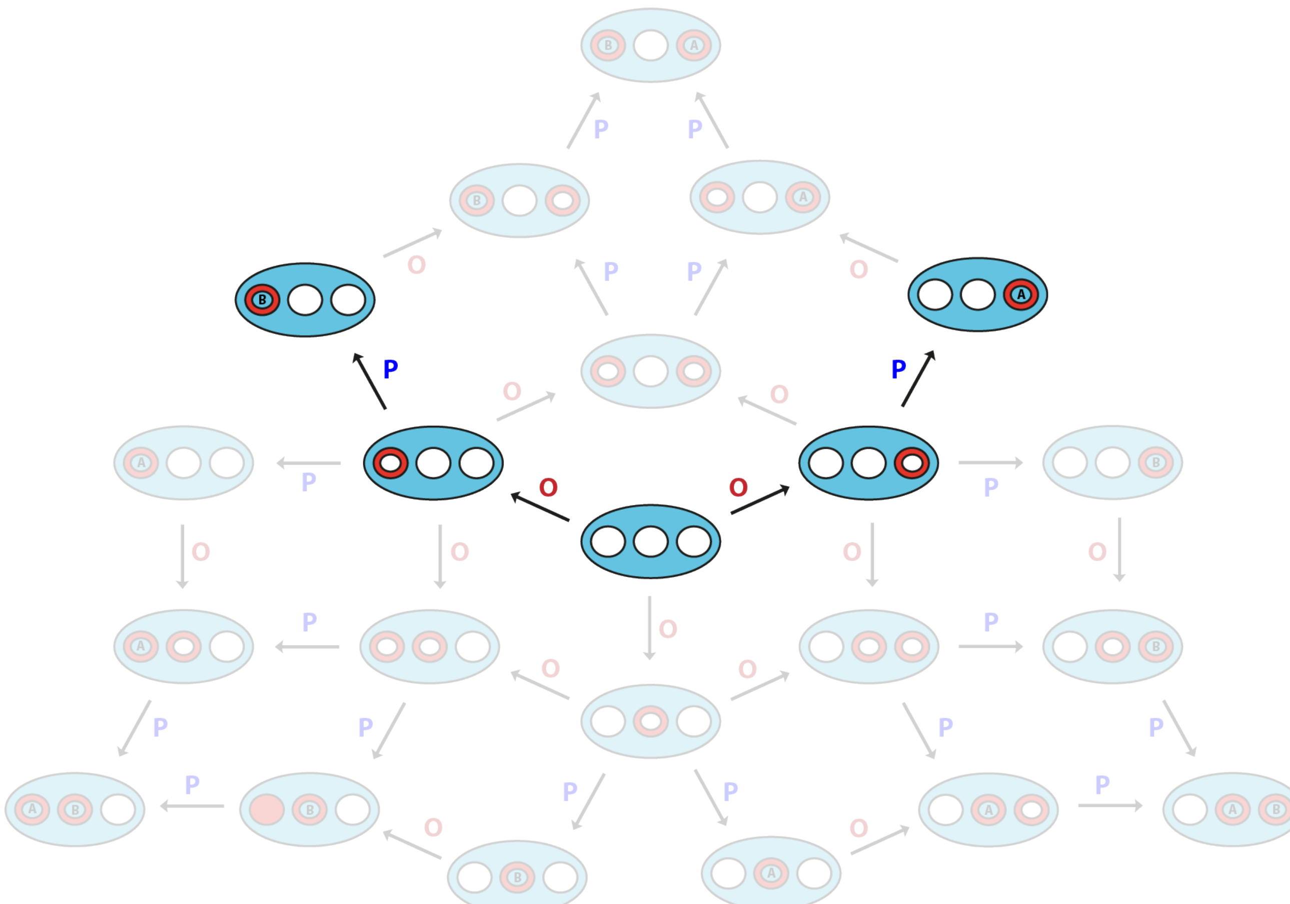
The Gustave formula in hypercoherence spaces



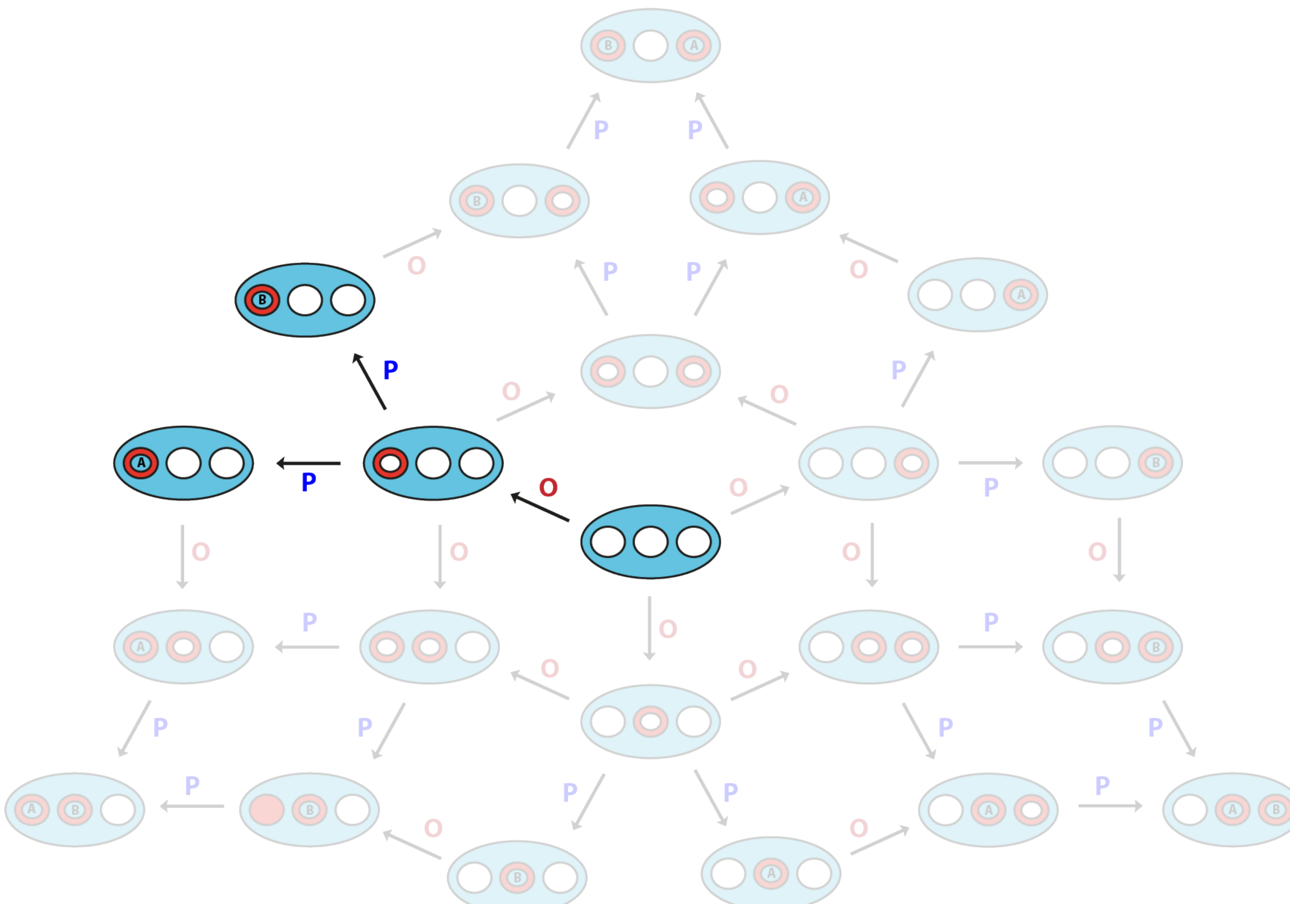
What hypercoherence says about the dialogue game



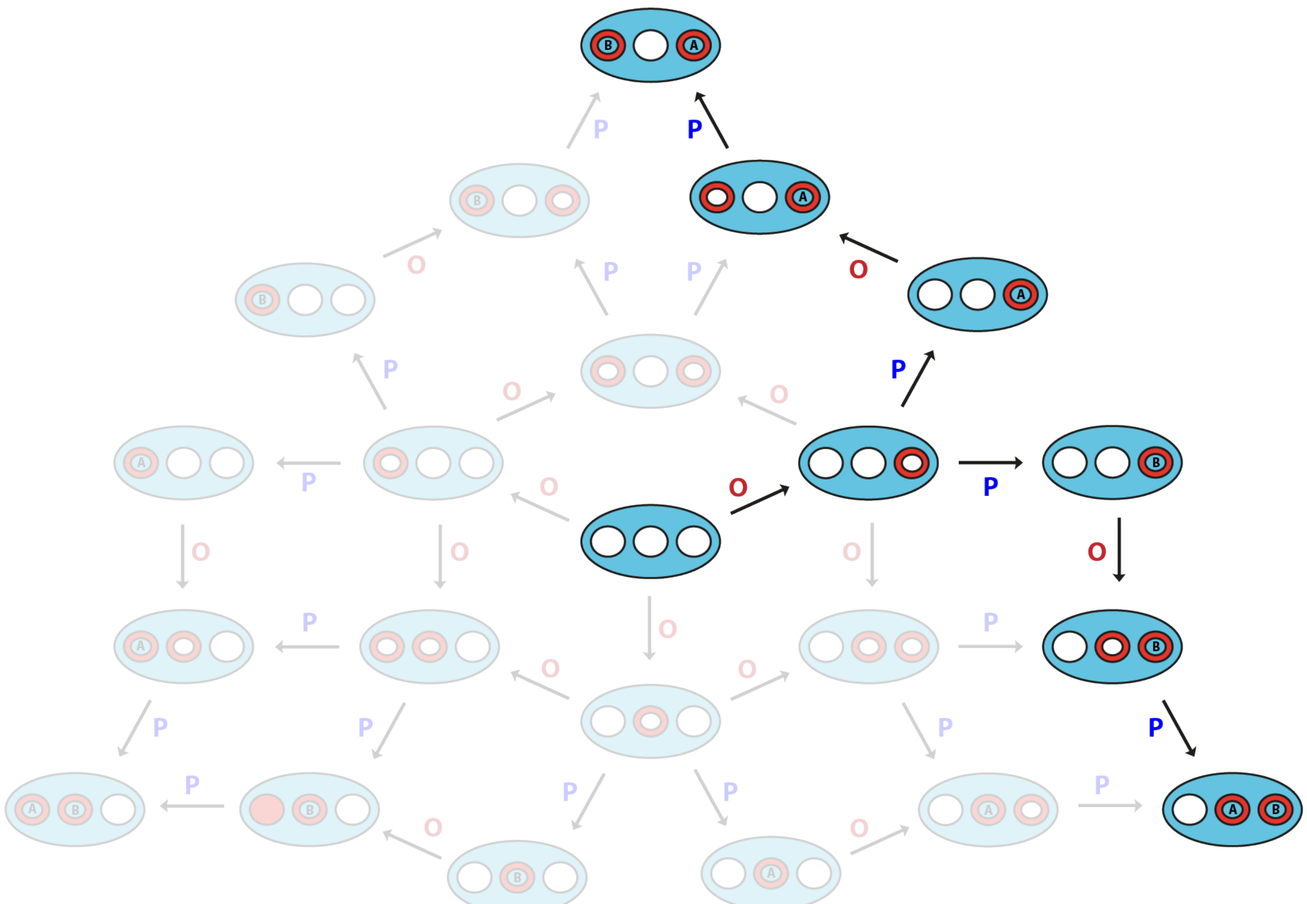
Here, the two positions are pairwise coherent



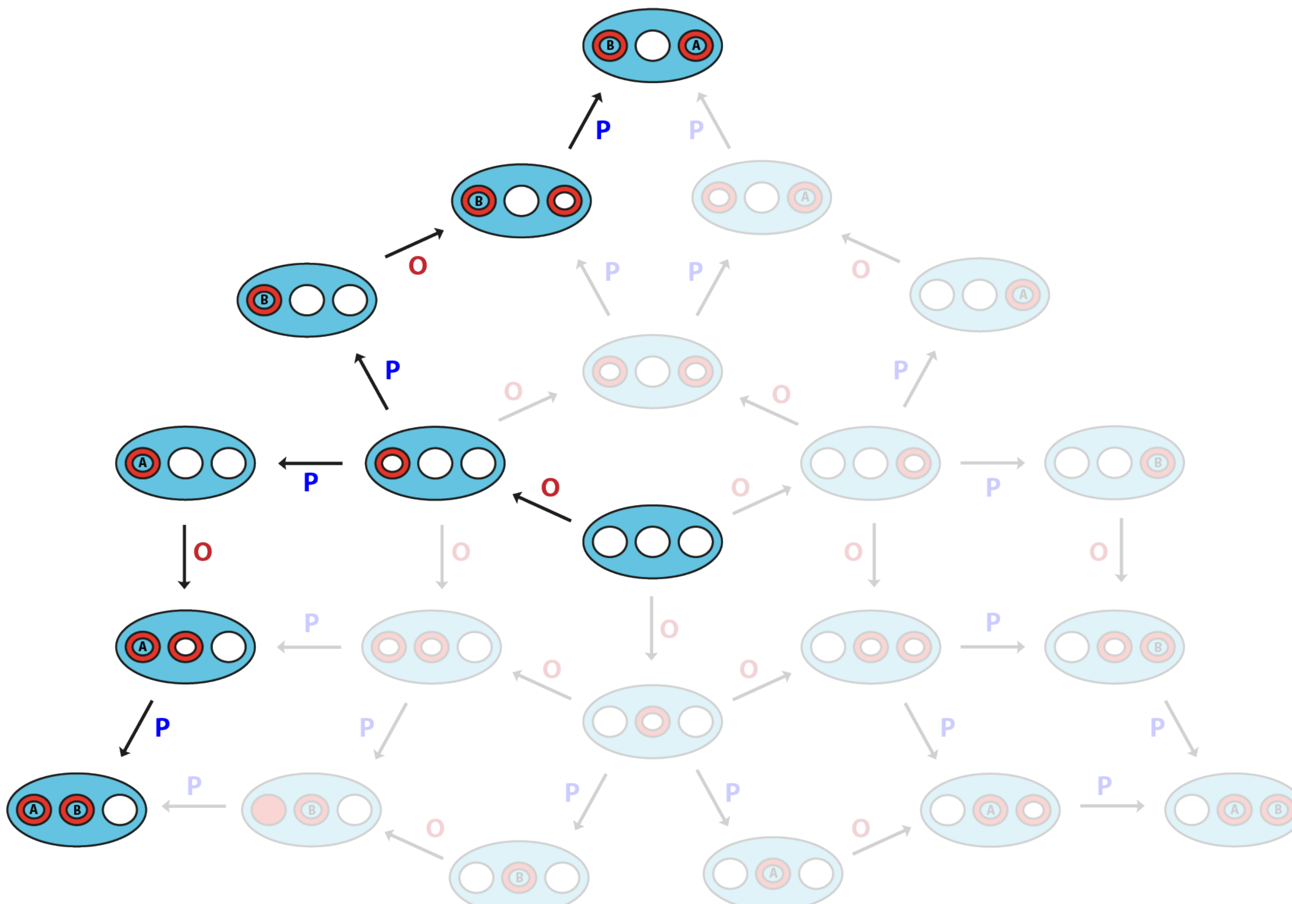
Here, the two positions are pairwise incoherent



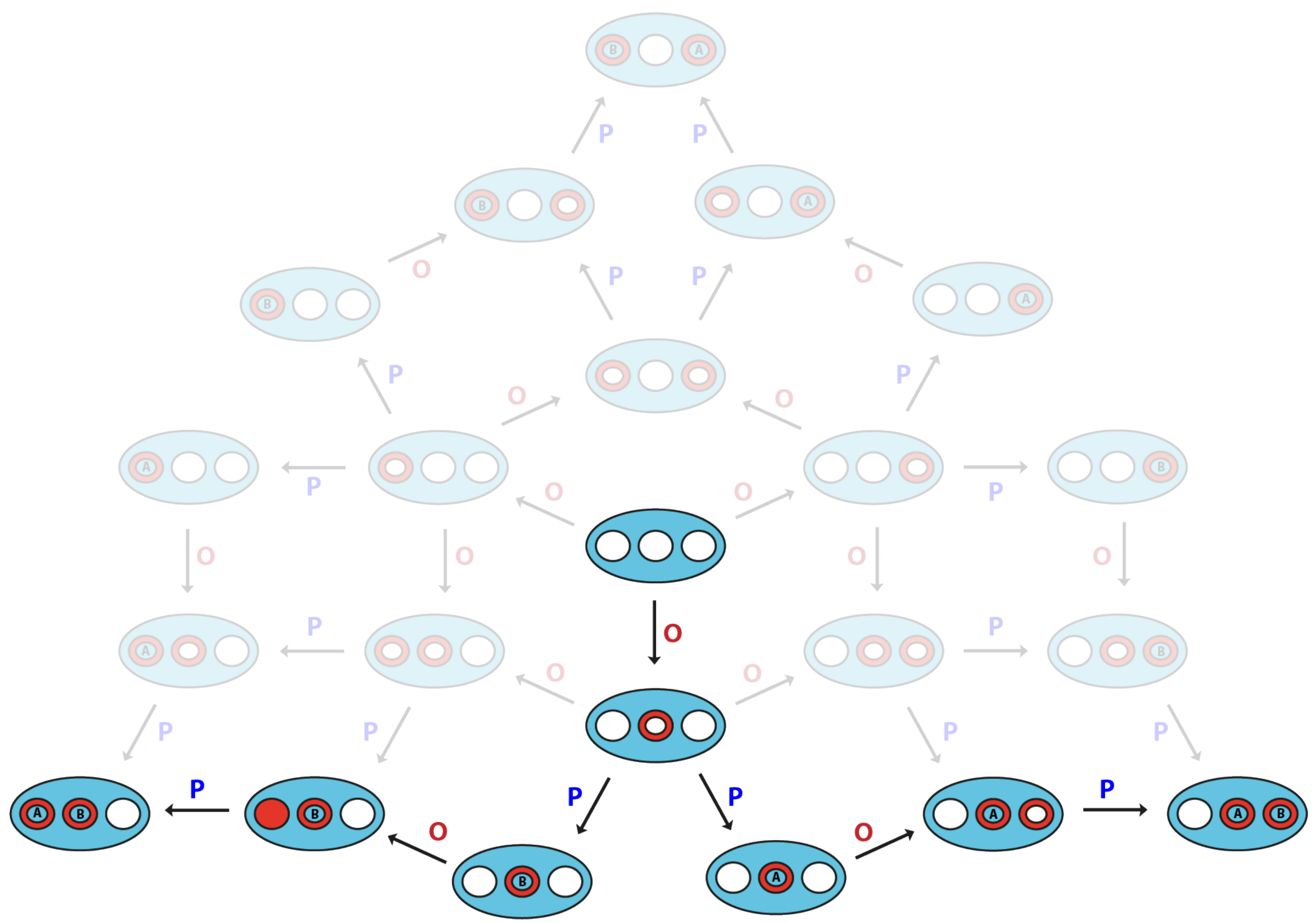
Gustave: the three positions are pairwise incoherent...



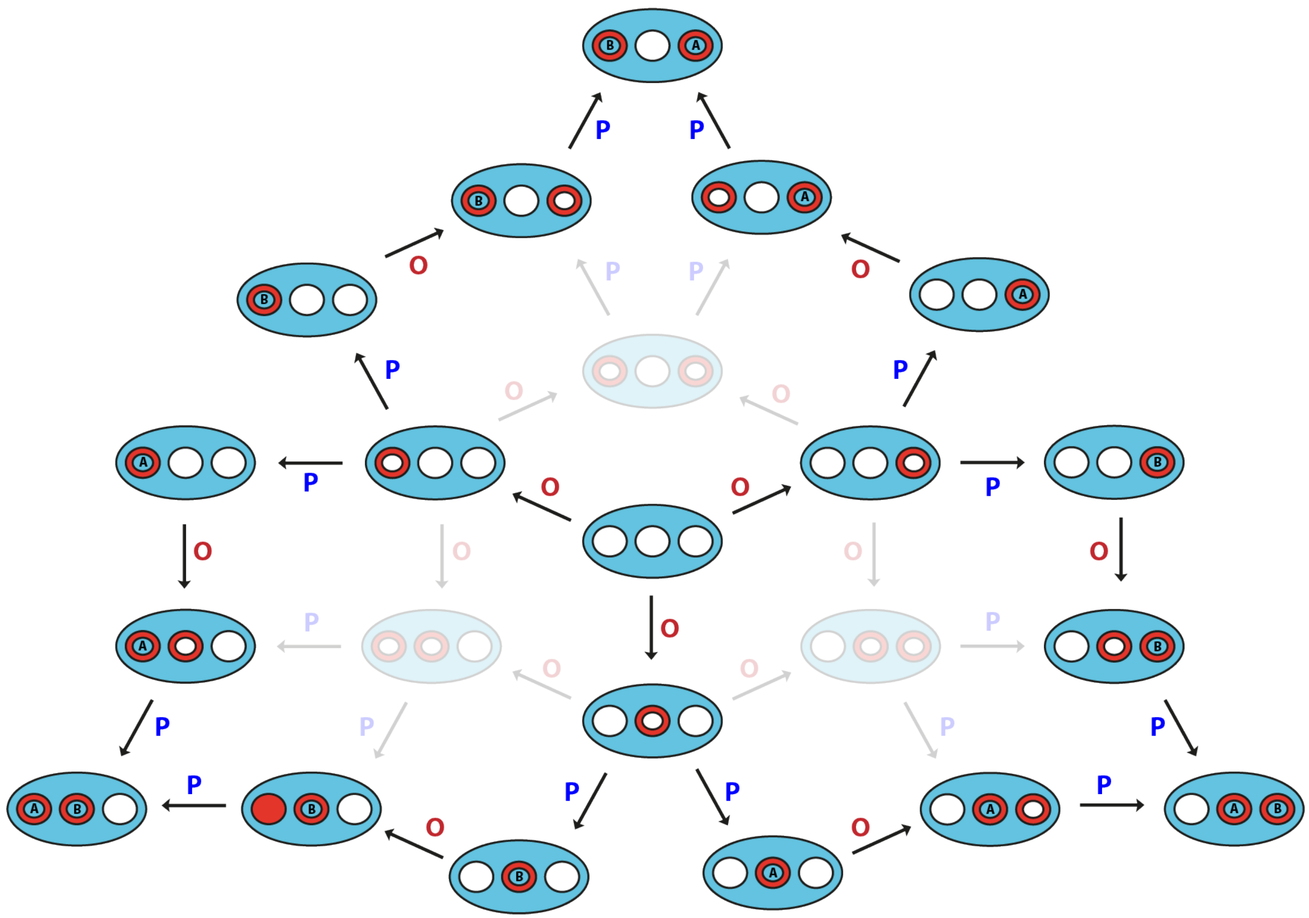
Gustave: the three positions are pairwise incoherent...



Gustave: the three positions are pairwise incoherent...



Gustave: the three positions are « threewise » coherent



Hypercoherence spaces and dialogue games

The beautiful idea by Ehrhard that strong stability has to do with

« higher-dimensional » forms of coherence

can be analysed in the framework of dialogue games.

Every finite set of positions v defines a **cone of sequential plays**

$$\mathbf{cone}(v) \quad = \quad \{ \quad s \in \mathbf{seqplay}_A \quad | \quad \forall x \in v, \; s \leq_A x \quad \}$$

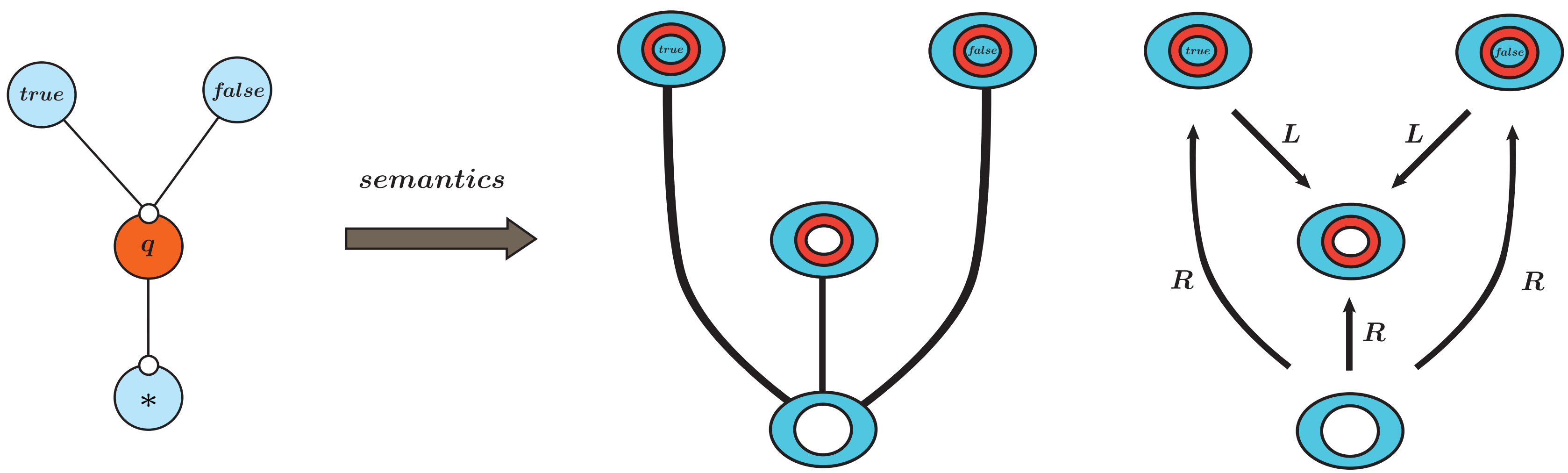
Then, it appears that in the intuitionistic fragment of linear logic, one has that every non-empty finite set of maximal positions is

- ▷ **coherent** precisely when Opponent can always play in the cone
- ▷ **incoherent** precisely when Player can always play in the cone

This basic observation leads (after some work) to another proof of Ehrhard's extensional collapse theorem. More details about the proof in Sequential algorithms and strongly stable functions, TCS 2005.

What about extensional stability ?

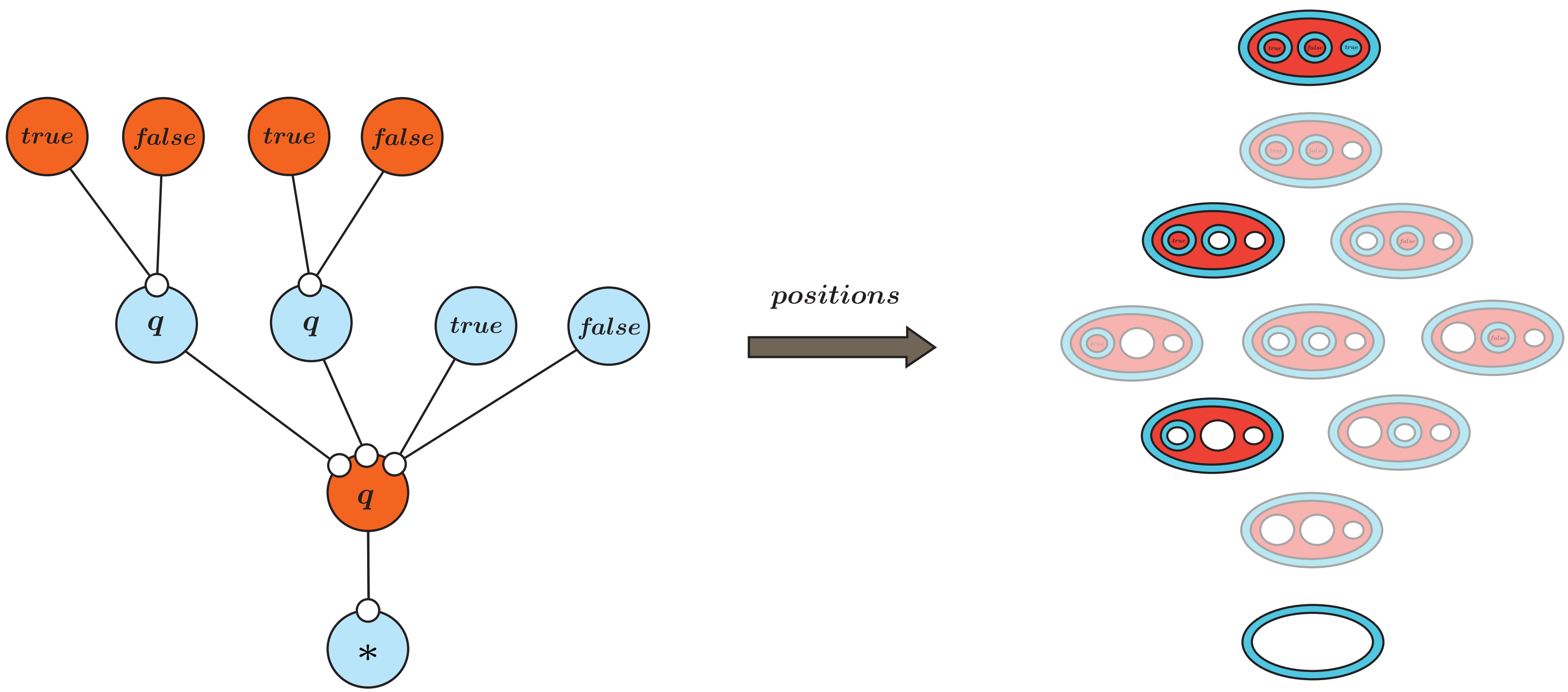
Illustration



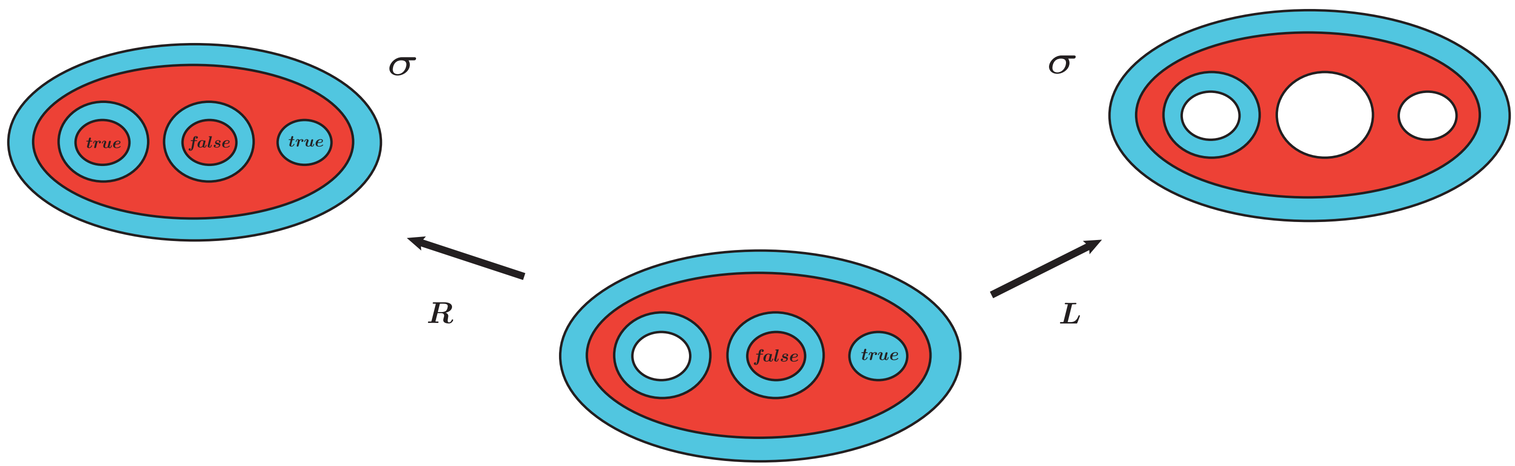
$\leq_R$ adds Opponent information

$\leq_L$ removes Player information

Left-to-right strategy



Extensionality in bistructures



A form of **retrospective game** :

Every time **Opponent** removes some information from a halting position, **Player** can get back into the strategy by removing his own information.

In a nutshell

Strong stability provides an elegant and concise description of the forward and sequential dynamics of proofs.

Extensional stability provides an elegant and interactive description of the backward and retrospective dynamics of proofs.

Thomas Ehrhard Parallel and serial hypercoherences. Theoretical Computer Science 2000	Pierre Boudes Projecting Games on Hypercoherences. Proceedings of ICALP 2004

A nice and flourishing field today

Layered object-based semantics for low level code

Arthur Oliveira Vale, PAM, Zhong Shao, Jérémie Koenig, Léo Stefanesco.
Layered and object-based game semantics.
Proceedings of POPL 2022

Concurrent separation logic

PAM and Léo Stefanesco.
Concurrent Separation Logic Meets Template Games.
Proceedings of LICS 2020

Homotopy models of differential linear logic

PAM, Template games and differential linear logic.
Proceedings of LICS 2019

Pierre Clairambault and Simon Forest
The cartesian closed bicategory of thin spans
Talk at LHC in two weeks, 2022.

Thank you Thomas

for your inspiring work and guidance!